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AN INVESTIGATION OF THE REASONING PROCESSES EMPLOYED IN A FIRST COURSE IN DEMONSTRATIVE GEOMETRY BY ALBERTA HIGH SCHOOL STUDENTS

A DISSERTATION

SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES

IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR

THE DEGREE OF MASTER OF EDUCATION

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BY

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CHAPTER I

DEMONSTRATIVE GEOMETRY AND THE CURRICULUM

Geometry on the defence.

The right of demonstrative geometry to a place on the school curriculum has been strongly questioned many times during recent years. In the past it was claimed that a course in geometry not only developed a reasoning ability but had, as well, certain disciplinary values which could ill afford to be missed in any educational program. With educationists in doubt concerning the disciplinarian values of learning, and with the school population changing so that a very small percentage of students continue their studies to the point where geometry is applied, the supporters of geometry have been forced to re-examine their position.

The geometry of Euclid as taught to the parents of our students, with its emphasis on formal proofs, constructions, theorems and rote learning, finds few proponents today. Since then school geometry appears to have branched in at least two directions. The first influenced by the progressive education philosophy emphasizes activity and utility. To have the students meet and find in the problems they face, situations which call for geometric knowledge and investigation is its aim. The problems may arise on the farm, at the carpenter's bench, in the shop of the mechanic or around the home. Utility, not logic, is the principle used to

determine the choice of material. The value of the geometry does not depend on the flimsy hope of transfer, it advocates claim; here the knowledge is directly applicable. Added to this advantage of immediate utility, is the claim that students interest becomes immediately aroused by material that has meaning to his daily living. With this enlivened interest the student may also learn the theoretical implications and generalizations without the feeling that he is being forced to learn something abstract and artificial.

A second plan in geometry teaching seeks to use geometry as subject matter by means of which to develop, on the part of the student, an ability to reason logically. There is no doubt that geometry exhibits a logical structure. Indeed Euclid's "Elements of Geometry" was man's first great epic of reasoning, the earliest work in which mathematical ideas were exhibited as a logically arranged sequence of propositions. However, Euclid's Elements were written for men, not for boys and girls; and text writing is still under the sway of Euclid's style. This formal logical emphasis may hinder the pupil's understanding of logical principles. Can geometry be taught in such a manner as to develop in the student a working knowledge of the laws of elementary logic? Can he be made more alert to the fallacious forms argument can take? If the student can be taught to be logical in the field of geometry, and if this teaching will carry over into other fields, then geometry teaching need offer no excuses. Through it, the student has gained the ability to reason, an ability justifying

strenuous effort. Recent results indicate that the "errors due to not understanding the meaning of proof, persist and account for the largest number of errors committed,"[1] and should warn us against assuming that a study of geometry means a mastery of logic. Consequently the hope that geometry taught with the ability to reason logically as an objective, can achieve the desired results must be justified.

In Alberta, the introduction of a new text in September 1940,[2] focused attention on these considerations concerning the teaching of geometry. While throughout the body of the text continual reference is made to, and use is made of every-day occurences of geometric problems, still the text is definitely prepared so as to develop an insight into the nature of logical reasoning. The author states in the preface,

"as the student begins to gain some appreciation of how his proofs rest fundamentally on the use of undefined words and unproved propositions, his earlier confidence may be somewhat shaken. But proofs do make the propositions hang together; there is a system and order, even if the system is not rigid and the order is not irrevocable. Indeed the student should begin to understand that theories and systems are not certainties or absolutes, but tools in human hands which may be used for the furtherance or hindrance of human welfare."

In short, then the text on which the Alberta curriculum is based aims to use the everyday situation approach with its advantage of utility and at the same time develop an ability to reason logically, an ability pre-requisite to the continuance of our democratic form of life. This approach is

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1. Rolland R.Smith-- Three Major Difficulties in Teaching of Demonstrative Geometry, reported in the Math. Teacher, March 1940.

2. Geometry for Today, A.J.Cook, Macmillan Co., Toronto, 1940.

directly in line with the aim suggested in the 13th yearbook of the N.C.T.M.: "Geometry achieves its highest possibilities if, in addition to its direct and practical usefulness, it can establish a pattern of reasoning; if it can develop the power to think clearly in non-geometric situations; if it can develop the power to generalize with caution from specific cases and to realize the force and all inclusiveness of deductive statements; if it can develop an appreciation of the place and function of definitions and postulates in the proof of any conclusion, geometric or non-geometric; if it can develop an attitude of mind which tends always to analyze situations, to understand their inter-relationships, to question hasty conclusions, to express clearly, precisely and accurately non-geometric as well as geometric ideas."[1]

In developing the power to think clearly the author of our present text is alert to use the fact that the pupil has already reasoned accurately on a variety of subjects. Basing his development on this already existing ability, the author attempts to mould the student's reasoning into more formal style. Nor are the exercises in reasoning confined to geometric situations. The student is encouraged to apply the same analytical and critical methods to non-geometric situations.

Purpose of Investigation.

This study is an attempt to determine if any relation exists between a student's ability in geometry and his mastery of logic. When reference is made to a student's ability in geometry it will be his ability to solve actual specific problems that will be meant. Is this ability in any way related to his grasp of that "certain form of reasoning, a method of attack that can be used on most problems."[2]

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1. The Nature of Proof-- 13th Year Book of the National Council of Teachers of Mathematics.

2. S.Clarke-- A.T.A. September 1940.

While the above query was the primary concern of this study, it was hoped that many interesting points would be revealed regarding the new Alberta curriculum. How does the present Alberta curriculum compare with the traditional curriculum? With the curriculum of other provinces? Will certain schools consistently do work above the average? Or if a school stresses logic will it do poorly on the more specifically geometric problems, especially those requiring skill in original geometric manipulation? This last question is practically the same question raised in the N.C.T.M. 13th year book. Using the terminology of this source the questions would read:- Can geometry achieve "direct and practical usefulness" and at the same time give the pupil "certain ideas about the nature of proof?"[1]

Specific information might also be found concerning the types of problems offering greatest difficulty, along with the reason for these difficulties, thus making the test act in a diagnostic capacity. Added to this is the possibility that an analysis of the results of schools in conjunction with the teacher questionaire, might throw light on teacher methods giving best results. In studying teacher methods, caution must be exercised not only because of the impossibility of measuring or controlling human qualities involved but also because of the countless other factors that might affect a school's attainment on the particular tests used.

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1. 13th Year Book N.C.T.M.

CHAPTER II

THE TEST BATTERY

Description of the Test Battery.

The test battery consists of five tests on geometry and reasoning, accompanied by the Otis B Higher Examination of Mental Ability. Since the purpose of the study is to examine the relation between the student's ability to solve geometric problems, and the student's ability to reason logically whether formally or informally, the tests range by degrees from the type where the correct answer only is of importance, to the type where emphasis is placed almost exclusively on the form of reasoning. Each test is designed with a specific aim in view, an aim which to a large extent dictated the type of questions making up the test. A questionnaire was completed by each participating teacher. This served to indicate the aims and emphasis of instruction. A Record Sheet, a copy of which follows, was also obtained from each pupil. A description of each test follows.

GENERAL INFORMATION AND INSTRUCTIONS FOR ADMINISTERING TESTS

eneral Statement

The study in which you are cooperating is designed to determine to hat extent our present courses in Demonstrative Geometry, as taught, evelop the student's ability to reason. A number of allied questions ill also be investigated at the same time. The research is sponsored the Canadian Council for Educational Research and is being conducted a committee consisting of Dr. A. J. Cook, Mr. Stanley Clarke, M.A., nd Mr. Leonard Pallesen, B.Sc. The scope of the study includes six rovinces: Nova Scotia, Quebec, Ontario, Manitoba, Alberta and British olumbia.

At the present time such a study is of considerable importance. e results are not intended to answer any academic question about ransfer of training in general, but rather to throw some light on ctual results being achieved at the moment. There is no intention of rawing invidious comparisons between results from various provinces.

It was considered necessary to obtain some information from each articipating teacher. For this purpose you will find a questionnaire hich you are requested to fill in immediately. Any additional comments ou wish to make will be welcomed. They can be attached to the uestionnaire.

ests

A test battery consisting of five tests on geometry and reasoning, d an Otis B. Higher Examination of Mental Ability, will be provided. is requires a total of approximately five hours of pupil time for mpletion. Three tests require about one hour each (two class periods) ile the other three can be conveniently finished in one class period.

The tests must be given in the right order--number one first, mber two next, and so on. Also, since each test is related to the evious one, absolutely no class discussion of any test or test item ould take place until after the last test has been given. Here is e order and time:

Test I -- about 20 minutes
Test II -- about 60 "
Test III -- about 60 "
Test IV -- In two parts, each about 20 minutes
Test V -- about 60 minutes

time limit is not necessary for any of the above tests. If possible, low sufficient time for each pupil to finish each test. The Otis B. telligence must also be given, even though you already have the I.Q. each pupil concerned, because (1) We want to use mental age, not I.Q., d (2) For comparable results on a large group the same test results st be used. You will be provided with an answer key so that you may cure the I.Q. scores if you wish.

[...]ministering the Tests

General -- You are requested to emphasize to the students not to mark [...] the test booklets where so instructed.

If possible, do not hurry students for any test of I-V, if sufficient [...]me can be allowed for completion by each student of each test.

Foolscap or scribbling paper may be used by the student for every [...]st except number three and the Otis B.

Test I -- Distribute the answer sheets first and have the students [...]ll out the information asked for at the top of the sheet. You will have [...] help them fill in their standing in class.

Then distribute the test. Go over the sample problem with the students [...]d make sure that students are recording their answers on the answer-sheet [...]rrectly. Avoid answering questions. If the student understands the [...]rections, let him go ahead.

Test II -- Distribute the answer sheets and have the student fill out [...]e information asked for. Distribute the sheet of reasons ($8\frac{1}{2}$ X 14 sheet). [...]en distribute the test. Go over the directions and the sample problem [...]th the students. Make sure they are recording their answers correctly on [...]e answer sheet.

Test III -- Distribute the same sheet of reasons used for Test II. [...] answer sheet is provided here. The answers are to be placed on the test [...]eets in the spaces provided. The student is to follow the general [...]rections for each problem. See that reasons are recorded in steps in [...]e blocked spaces provided.

Test IV -- The first part (the sheet of statements to be marked T or [...] must be given first and the sheets collected before the second part is [...]arted. After giving part one, if there is sufficient time to do part two [...] the same class period, distribute the answer sheet first, then the test, [...] before. Have the student note that some blanks require a word or phrase, [...]me a number or numbers, according to the sense of the question. See that [...]ey are being recorded correctly on the answer sheet.

Test V -- Follow the same procedure about distributing the answer [...]eets, then the test booklet. Read aloud the directions and sample prob-[...]m and definitions, with the pupils following you in their booklets. It [...] essential that this procedure be followed, since this is a new type of [...]st in the battery, and the student must understand how it is done before [...] tries to proceed. Tell the students to consider each problem carefully [...]d not to hurry too fast.

Otis B. Test -- Follow the instructions given in the manual. It [...] self administering.

[...]w to Return the Tests

If you receive more copies than you require, please return those not [...]quired immediately. For the rest, wait until all the tests are completed [...]hich will require about two weeks), then return all the test booklets, [...]e answer sheets, Otis B. tests and questionnaire (everything except your [...]mplimentary copies), express collect to:

Dr. M. E. LaZerte,
Principal, College of Education, University of Alberta,
Edmonton, Alberta.

You will probably find the tests interesting and valuable. For [...]arly all the tests (all except number III and the first part of number [...]), you can redistribute the question booklets after all the tests have [...]en completed, and discuss the test items. In addition, you may keep a [...]mplimentary copy of each test (except the Otis B.)

RECORD SHEET

ne: ______________________ School: ______________ ____

ar in High School: __________ Grade IX standing (A,B, or C) ______

Subjects you have taken last year or before please check (✓)

Subjects you are taking this year please mark with a cross (X)

(check or cross)

English I ____

Social Studies I ____

Algebra I ____

Geometry I ____

Physics I ____

Latin I ____

French I ____

Chemistry I ____

(check or cross)

English 2 ____

Social Studies 2 ____

Chemistry 2 ____

Physics 2 ____

Latin 2 ____

French 2 ____

List here the other subjects you have taken:

______________________ 6. ______________________

______________________ 7. ______________________

______________________ 8. ______________________

______________________ 9. ______________________

______________________ 10. ______________________

List here the other subjects you are taking:

______________________ 6. ______________________

______________________ 7. ______________________

______________________ 8. ______________________

______________________ 9. ______________________

______________________ 10. ______________________

Test I.- The questions in Test I call for only answers. The material is elementary, consisting of simple exercises in similarity, congruence, parallelism, and the basic properties of triangles and parallelograms. It was expected that the material of the test would be familiar to all candidates who had studied geometry. Hence if an exercise was missed, the omission would likely be due to lack of ability to apply this knowledge. Naturally then it was expected that the scores be high. Since no attempt was made to record the method of problem attack used by the pupil, the pupil's score on the sixteen problems of the test may thus be considered only in the nature of an index of his ability to "get answers" irrespective of the rationality of the method used.

To facilitate marking and to make possible the future use of the test itself an "Answer Sheet" was provided of which the pupil was requested to place his answers in the indicated blanks.

ANSWER SHEET TEST I

ame: ______________ School: ______________

ge: ____(yrs.) _____(months) City or Town: ______________

rade: ________ Sex: ________ Geometry Teacher: ______________

umber of years you have been attending High School (including this year): ______

our standing in class in geometry course you are taking: Check below.
____(lowest 1/3) ____(highest 1/3) ____(middle 1/3)

umber of courses in Demonstrative Geometry you have taken (including courses you are taking this year): ________

Sample Problem

(1) 62

Exercise 1

(1) ________

Exercise 2

(2) ________

Exercise 3

(3) ________

Exercise 4

(4) ________

(5) ________

(6) ________

Exercise 5

(7) ________

Exercise 6

(8) ________

Exercise 7

(9) ________

(10) ________

Exercise 8

(11) ________

(12) ________

Exercise 9

(13) ________

(14) ________

(15) ________

Exercise 10

(16) ________

TEST I

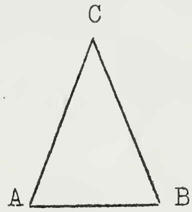
rections: Study the following figures and the given facts, then compute the answers, which may be left in the radical form or worked to two decimal places. Record your answers on the answer-sheet provided. Do not place any marks on this sheet.

Sample Problem

In triangle ABC, CA = BC

$\angle$ CAB = 62°

Then $\angle$ CBA = (1) °

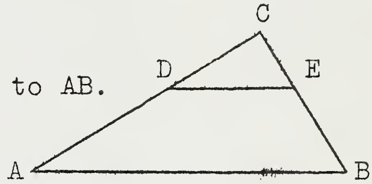


cording your answer: On the answer sheet opposite (1), place the correct answer. Thus (1) 62

Exercise 1

In triangle ABC, DE is parallel to AB.
CD = 3, AD = 5, CE = 2.

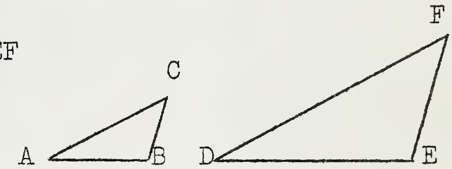
Then EB = (1)



Exercise 2

Triangle ABC is similar to triangle DEF
AB = 6, BC = 4, AC = 8, DE = 12

Then EF = (2)

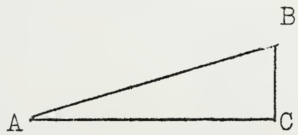


Exercise 3

ABC is a right triangle

AC = 24, BC = 7

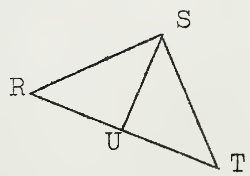
AB = (3)



Exercise 4

RST is an isosceles right triangle
SU is perpendicular to RT
RT = 20 inches, RS = (4) inches.

SU = (5) inches. $\angle$ UST = (6) °

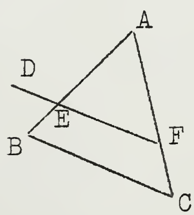


Exercise 5

DF is parallel to BC

$\angle$DEA = 111°, $\angle$BCF = 53°

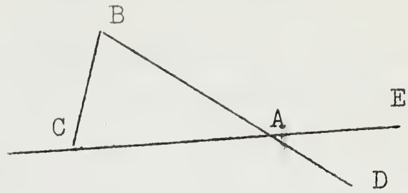
$\angle$ BAC = (7) °



Exercise 6

AB = AC

$\angle DAE = 36^\circ$ $\angle CBA =$ ___(8)___ $^\circ$



Exercise 7

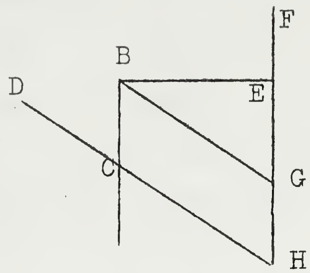
BG is parallel to DH
BC is parallel to HF

$\angle BCD = 57^\circ$

$\angle BEF = 90^\circ$

$\angle BGH =$ ___(9)___ $^\circ$

$\angle GBE =$ ___(10)___ $^\circ$



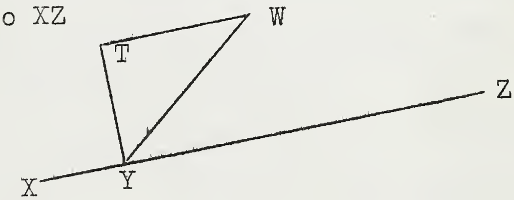
Exercise 8

TY is perpendicular to XZ
TW is parallel to XZ

$\angle XYW = 141^\circ$

$\angle TYW =$ ___(11)___ $^\circ$

$\angle TWY =$ ___(12)___ $^\circ$



Exercise 9

The chord RS is perpendicular
to the diameter AB

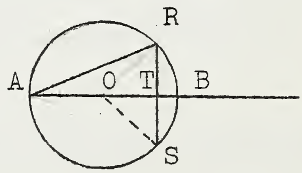
RS = 6 cm.

$\angle RAB = 22.5^\circ$

RT = ___(13)___ cm.

$\angle SOT =$ ___(14)___ $^\circ$

OT = ___(15)___ cm.



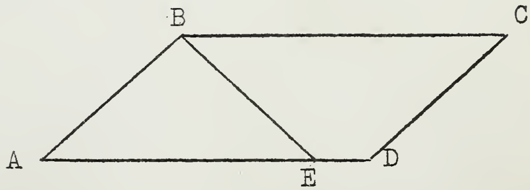
Exercise 10

ABCD is a parallelogram

$\angle A = 42^\circ$

AB = BE

$\angle EBC =$ ___(16)___ $^\circ$



Test II.- In Test II an attempt was made to determine the student's ability to follow and to supply steps in a logically developed solution. As in Test I, the basic geometric knowledge required for the handling of the problems is kept at a minimum. in fact, the exercises of Test II can almost completely be paired with exercises of Test I requiring the same geometric concept.[1] It was thought that this might prove useful in diagnostic investigations. In all, Test II had seventy-nine blanks for completion, the answers being made on an Answer Sheet.

The general outline of a solution to each problem was given in two columns headed Statements and Reasons. The responses sought can be classified as follows:

(a) In some cases the student was required to select from a given list of reasons the supporting reason for a statement made.

(b) The application of theorems to specific problems was checked by requiring the student to show the implication in the problem of a particular theorem whose number was inserted in the Reasons Column.

(c) The more difficult task of giving both the statement needed in the proof and the supporting reason was required in some of the questions of the test.

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1. See Chapter V Table XIV Page 84.

Before any conclusions are drawn from this test, it must be pointed out that those conclusions will be based on the following assumptions:

(a) If the student can select from a list of 27 theorems the one supporting or justifying a given statement, then he must comprehend in this problem both the theorem and statement. He has then apparently begun to learn the technique of going from the particular to the general. i.e. he has learned to generalize.

(b) If a student can insert the reasoning step which the acceptance of a theorem makes necessary then too he understands, in this problem, the reverse of the above process. i.e. the argument from the general to particular.

(c) When he supplies both statement and reason he reveals that he is beginning to apply the technique if logical thinking in the solution of specific problems confronting him.

ANSWER SHEET TEST II

e: ______________________ School: ______________________

: _____(yrs.) _____(months) City or Town: ______________________

de: ____________ Sex: ____________ Geometry Teacher: ______________________

ber of years you have been attending High School (including this year) _____

r standing in class in geometry course you are taking: Check below.
_______(lowest 1/3) _______(highest 1/3) _______(middle 1/3)

ber of courses in Demonstrative Geometry you have taken (including
rses you are taking this year): _______

---

ple Problem

) g
) 55
) 125

ercise 1

) ________
) ________
) ________
) ________
) ________
) ________
) ________

ercise 2

) ________
) ________
) ________
) ________
) ________
) ________
) ________
) ________
) ________
) ________

Exercise 2(cont.)

(18) ________

Exercise 3

(19) ________
(20) ________
(21) ________
(22) ________
(23) ________
(24) ________
(25) ________
(26) ________
(27) ________

Exercise 4

(28) ________
(29) ________
(30) ________
(31) ________
(32) ________
(33) ________
(34) ________
(35) ________
(36) ________
(37) ________

Exercise 4(cont.)

(38) ________
(39) ________
(40) ________
(41) ________

Exercise 5

(42) ________
(43) ________
(44) ________
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(63) ________
(64) ________

Exercise 6

(65) ________
(66) ________
(67) ________
(68) ________
(69) ________
(70) ________
(71) ________
(72) ________
(73) ________
(74) ________
(75) ________
(76) ________
(77) ________
(78) ________
(79) ________

REASONS TO BE USED IN TESTS II & III.

(The reasons have been roughly grouped, for convenience.)

I. Things equal to the same thing are equal to each other.

II. Equals added to equals give equals.

Two Straight Lines.

III. If two straight lines intersect, the vertically opposite angles are equal.

IV. If two straight lines intersect, the sum of any two adjacent angles is 180 degrees.

V. If one line is perpendicular to another then the angles formed are each equal to 90 degrees.

VI. When two parallel straight lines are cut by a transversal, then the sum of any two interior angles on the same side of the transversal is 180 degrees.

VII. When two parallel straight lines are cut by a transversal, the corresponding angles are equal.

VIII. When two parallel straight lines are cut by a transversal, the alternate angles are equal.

The Triangle.

IX. The sum of the angles of a triangle is equal to 180 degrees.

X. If one angle of a triangle is 90 degrees then the other two angles are complementary.

XI. Any exterior angle of a triangle is equal to the sum of the two interior and opposite angles.

XII. Definition: A triangle which has two equal sides is isosceles.

XIII. If two sides of a triangle are equal, then the angles opposite the equal sides are equal.

XIV. If two angles of a triangle are equal, then the sides opposite the equal angles are equal.

XV. Definition: A triangle which has three sides equal is an equilateral triangle.

XVI. If three sides of a triangle are equal, then the angles are also equal.

XVII. The perpendicular from the vertex of an isosceles triangle to the opposite side bisects that side.

XVIII. The square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides.

Two Triangles.

XIX. If two sides and the included angle of one triangle are equal to two sides and the included angle of another triangle, then the two triangles are congruent.

XX. If three sides of one triangle are equal to three sides of another triangle, then the triangles are congruent.

XXI. If two angles and a side of one triangle are equal to two angles and a side of another triangle, then the two triangles are congruent.

XXII. If two triangles are congruent, then the corresponding sides and angles are equal.

XXIII. If two angles of one triangle are equal to two angles of another triangle, then the two triangles are similar.

XXIV. If two triangles are similar, then the corresponding sides are proportional.

The Circle.

XXV. All radii of a circle are of equal length.

XXVI. The perpendicular from the centre of a circle to a chord bisects the chord.

XXVII. The radius to the point of contact of a tangent to a circle is perpendicular to the tangent.

TEST II

ections: In the following exercises an outline is given of the steps essary to obtain the solution, but there are blanks left throughout you to fill.

The first column gives the steps necessary in the argument. The ber of the reason for each step (obtained from the list of reasons XXVII) is given in the second column.

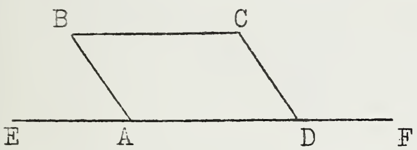
If there is a blank in the second column, you must find in your of reasons, the reason justifying the corresponding statement in first column. The number of this reason is to be recorded on the ver sheet. Should the statement be a statement of what is given, e the letter "g" in the numbered space on the answer sheet. If re is a blank in the first column, fill in on the answer sheet the ult that is obtained with the use of the reason whose number is shown he second column.

: As indicated above, all answers in test II are to be recorded on answer sheet. Do not put any marks on the test sheets.

Sample Problem

en The parallelogram ABCD with side AD produced to E and F. Angle EAB is 55°.

The angle CDF.



Statements	Reasons
(a) $\angle EAB = 55^\circ$	(1)
(b) $\angle CDA =$ (2)°	VII
(c) $\angle CDF =$ (3)°	IV

Comments

(a) Since statement (a) is part of the given data, the letter (g) has been inserted in space (1) of the answer sheet.

(b) Reason VII applied to this figure would give

$\angle CDA = \angle EAB = 55^\circ$. Hence 55 is shown in space (2) on the answer sheet.

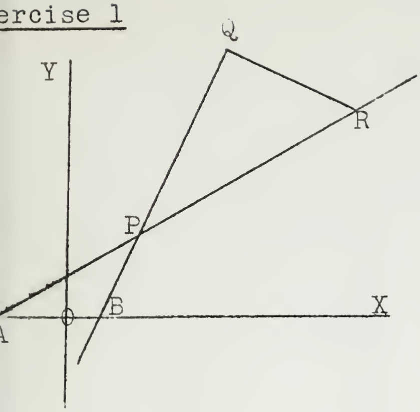
(c) Reason IV applied here shows that

$$\angle ADC + \angle CDF = 180^\circ$$

$$\text{But } \angle ADC = 55^\circ$$

Hence $\angle CDF = 125^\circ$, and 125 is shown in space (3) of the answer sheet.

ercise 1

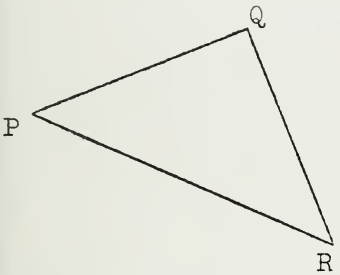


Given QR ⊥ to QP, $\angle$QRP = 55°, $\angle$PAX = 30°

Find $\angle$PBX

Statements	Reasons
(a) $\angle$QRP = (1)°	g
(b) ∴ $\angle$QPR = (2)°	X
(c) also $\angle$APB = (3)°	(4)
(d) $\angle$PAB = 30°	(5)
(e) ∴ $\angle$PBX = (6)°	(7)

ercise 2

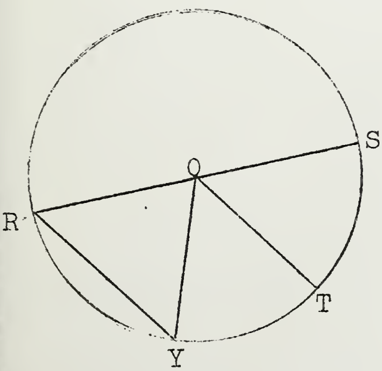


Given PQR is a right isosceles triangle with right angle at Q. The length of QR is 10 inches.

Find The length of PR and the angle R.

Statements	Reasons
(a) $\angle$Q = (8)°	g
(b) QR = (9) inches	g
(c) PQ = (10) inches	(11)
(d) ∴ RP = (12) inches	(13)
(e) $\angle$P + $\angle$R = (14)°	(15)
(f) but (16) = (17)	XIII
(g) ∴ $\angle$R = (18)°	

ercise 3

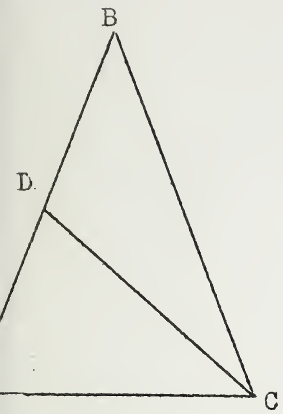


Given ROS is a diameter of the circle. OT is parallel to the chord RY.

Prove $\angle$TOY = $\angle$TOS

Statements	Reasons
(a) OR = OY	(19)
(b) ∴ (20) = $\angle$RYO	(21)
(c) also $\angle$RYO = (22)	VIII
(d) (23) = (24)	I
(e) (25) = (26)	VII
(f) ∴ $\angle$TOS = $\angle$TOY	(27)

ercise 4

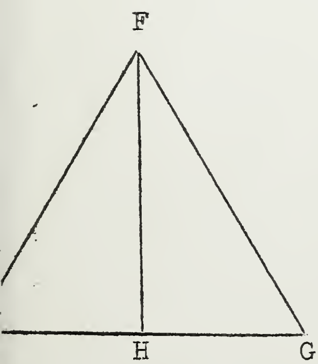


Given Triangle ABC with AB = BC.
D is on AB so that AC = DC.

Prove $\frac{AB}{AC} = \frac{AC}{AD}$

	Statements	Reasons
(a)	AB = BC	(28)
(b)	∴ (29) = (30)	(31)
(c)	(32) = (33)	g
(d)	∴ ∠CAD = ∠CDA	(34)
(e)	∴ (35) = (36)	I
(f)	also ∠CAD = ∠CAB	identity
(g)	∴ (37) is similar to (38)	XXIII
(h)	AB in △ACB corresponds to (39) in △ADC	
(i)	AC in △ACB corresponds to (40) in △ADC	
(j)	Hence $\frac{AB}{AC} = \frac{AC}{AD}$	(41)

ercise 5

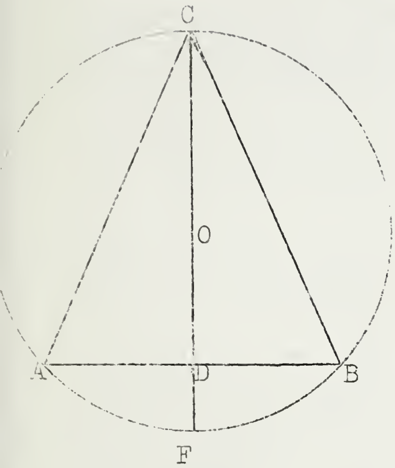


Given H is the midpoint of EG, the side of the equilateral △EFG. EG = 2a inches.

Find The length of FH and the angle HFG.

	Statements	Reasons
(a)	(42) = (43)	g
(b)	(44) = (45)	XV
(c)	(46) = (47)	identity
(d)	∴ (48) is congruent to (49)	(50)
(e)	Hence ∠FHG = ∠FHE	(51)
(f)	but (52) + (53) = (54)	IV
(g)	∴ ∠FHG = (55) °	
(h)	FG = (56) inches	(57)
(i)	HG = (58) inches	g
(j)	∴ FH = (59) inches	(60)
(k)	∠G = (61) °	IX and (62)
(l)	∴ ∠HFG = (63) °	(64)

xercise 6



Given COF is a diameter perpendicular to the chord AB.

Prove ACB is an isosceles triangle.

Statements	Reasons
(a) CD is perpendicular to AB	(65)
(b) $\therefore$ (66) = (67)	XXVI
(c) $\angle$ADC = (68) °	(69)
(d) $\angle$CDB = (70) °	(71)
(e) $\therefore$ (72) = (73)	I
(f) CD = CD	identity
(g) $\therefore$ (74) is congruent with (75)	(76)
(h) Hence $\triangle$CAB is isosceles since (77) = (78)	(79)

Test III.- In Test II where the pupil was required to give both his statement and the supporting reason, he was in reality supplying his own solution. Still, the general outline provided, gave incentive and help in the development of a formal solution. Test III requires the student to develop the entire proof or solution of the problem; both the steps in the argument and the reasons for each step were required. An attempt was made to base the problems on meaningful everyday situations yet involve only the geometric principles and theorems already used in Tests I and II. In fact, the same list of theorems supplied for Test II was used in Test III. Thus from the geometric point of view, Test III was no more difficult than the preceeding tests, but the students must have matured in thought to successfully attack it. Not only must he detect the basic geometric nature of an everyday problem, but also he must pick out the essential elements, what is given, what he is to find, and apply his knowledge of geometric facts to the development of a solution.

With a view to facilitating marking, a framework was supplied within which the student was required to place his solution. The test consists of five exercises.

TEST III

rections: In the following exercises, draw a freehand figure and letter
e figure. Show what is given along side of the figure. After the style
test II, state the main steps (a), (b), ... in your argument, giving
e reason for each step. Use the list of reasons provided for test II.

You may not, of course, require all the spaces (a)....(k) for steps
the argument. Only one step is to be shown in each space.

Exercise 1

roof gable is in the form of a right triangle, the right angle being at
e apex of the gable. If the base of the gable is 12 feet, find
) the height, (ii) the slant edge of the gable.

Given (in terms of the lettered figure):-

(1)

(2)

(3)

Figure.
etter apex A and base BC.)

eps in the argument.	Reasons
a)	
b)	
c)	
d)	
e)	
f)	
g)	
h)	
i)	
j)	
k)	

(i) The height is ________ ft.

(ii)The slant edge is ________ ft.

ate the definition of a right isosceles triangle:- ____________________

____

____

Exercise 2

anti-aircraft battery can fire a distance of 20,000 ft., i.e. the tery has a range of 20,000 ft.. An enemy plane is flying at a constant ght of 12,000 ft.. Over what distance is the plane in the range of battery?

Given (in terms of the lettered figure):-

(1)

(2)

(3)

Figure.
tter the battery B, and the path of the plane MN.)

Steps in the argument.	Reasons
)	
)	

refore the distance the plane is in the range of the battery is

____________ feet.

Exercise 3

ree radio stations A, B and C are situated such that the line AB is degrees north of the easterly direction, the line AC is 76 degrees rth of the easterly direction, and the line BC is 24 degrees west of northerly direction. Calculate the angle ACB.

Given (in terms of the lettered figure):-

(1)

(2)

(3)

(4)

(5)

Figure.
ot the north-south lines and
e east-west lines through
ch station B and C.)

Steps in the argument. | Reasons

a)

b)

c)

d)

e)

f)

g)

)

).

)

)

Therefore the angle is degrees.

Exercise 4

ighway curve is circular in shape. A fence 1000 ft. long runs straight
the inside of the curve to adjoin the curve at each end of the fence.
perpendicular distance from the centre of the curve to the fence is
00 ft. Find the radius of the highway curve.

Given (in terms of the lettered figure):-

(1)

(2)

(3)

(4)

(5)

(6)

Figure.
bel the centre O, the fence
, and the perpendicular OD.)

Steps in the argument.	Reasons
a)	
b)	
c)	
d)	
e)	
)	
)	
)	
)	
)	
)	

refore the radius of the circle is ____________ feet.

Exercise 5

shelf for the corner of a room is to be constructed in the form of a ght isosceles triangle. It is to hold a radio of dimensions 8 inches 12 inches. The front of the radio is to be flush with the hypotenuse the triangle shelf. Find the dimensions of the shelf.

Given (in terms of the lettered figure):-

(1)

(2)

(3)

(4)

(5)

(6)

Figure.

C is the hypotenuse of the shelf ABC, and D and E are the front rners of the radio DEFG.)

Steps in the argument.	Reasons
ı)	
)	
)	
)	
)	
)	
)	
)	
)	
)	
)	

refore the sides of the triangle are:

_________ inches, _________ inches, and _________ inches.

Test IV, Part I.- This test gives thirty-four statements of the principles of logic and of the fallacies that might arise in reasoning. These statements are arranged in pairs so that one member of each pair is true while the other is false. The statements are not made in geometric terminology but as examples of principles of logic that might be mastered in a course in geometry. The principles in each pair are arranged with slightly different wording. In this way, the recognition by the student of principles learned by rote, but otherwise meaningless, is discouraged. Almost certainly this ensures that a correct response indicates grasp of the full meaning of the principle. Student success on this test will serve as an index of his knowledge of certain general principles fundamental to logical reasoning such as the use of undefined words and unproved propositions. Unfortunately the results will throw no light on the question as to whether his knowledge has been gained from the study of geometry, from the study of some other subject, or merely incidentally. The following are the pairs of True-False statements:- 1 and 21, 5 and 33, 8 and 16, 2 and 24, 9 and 23, 3 and 32, 18 and 30, 14 and 26, 15 and 27, 4 and 13, 11 and 22, 6 and 17, 7 and 29, 10 and 31, 12 and 19, and 25 and 34.

TEST IV Part I

Some of the statements which follow are true, others are false. In the space to the left of each item mark the item true (T) or false (F).

_ 1. There is only one possible definition for a word.

_ 2. In an argument every important word should be agreed upon or defined if the same conclusion is to be reached by all.

_ 3. To obtain a conclusion in an indirect argument, all the possibilities must be considered and all but one of these possibilities must be eliminated.

_ 4. One can prove the lack of all merit in a person or institution by demonstrating the lack of merit in certain aspects of the person or institution.

_ 5. Changing the meaning of one of the words in the statement of a definition has the effect of changing the definition.

_ 6. If a statement is true then the converse of that statement is also true.

_ 7. A false statement used as a reason may lead to a wrong conclusion.

_ 8. A good dictionary will always settle any dispute about the particular definition of a word to use.

_ 9. When the people concerned are agreed about the meaning of a word, a correct conclusion can be reached without a formal definition of the word.

_ 10. A man who is expert in some subject will be able to reason logically in any discussion.

_ 11. A logical argument cannot be disproved by ridiculing the arguer , or his motives, or by attacking his motives.

_ 12. Any true statement can be used in support of a given argument.

_ 13. An attack upon certain aspects of a person or institution, even though justified, is not sufficient to prove the lack of all merit in that person or institution.

_ 14. A false assumption may lead to a false conclusion.

_ 15. If an argument contains an unproved proposition or assumption, then it cannot be valid.

_ 16. Since a dictionary usually gives several possible definitions of a word, it does not always settle all disputes about the meaning of a word.

_ 17. The converse of a true statement is not always true.

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_ 15. If an argument contains an unproved proposition or assumption, then it cannot be valid.

_ 16. Since a dictionary usually gives several possible definitions of a word, it does not always settle all disputes about the meaning of a word.

_ 17. The converse of a true statement is not always true.

18. If there are three and only three possibilities and one of them must happen, then if two of the possibilities are shown to be in fact impossible, then the third must happen.

19. A true statement must be relevant (i.e. applicable to the given situation) if it is to be used in logical argument.

20. If one changes a definition, the conclusion which follows may also be changed even though the argument is logical.

21. There may be several defintions for a word.

22. Many arguments are logically proved by showing that the opponent's motives are unworthy or ridiculous.

23. Even though those concerned are agreed about the meaning of a word in an argument, it is necessary to define it formally in order to reach a correct conclusion.

24. A problem has only one solution even if all the important words in its statement are not agreed upon or defined.

25. In any argument what is to be proved should not, either directly or indirectly, be used as part of the argument.

26. Even if an assumption is false, logical reasoning from this assumption will lead to a correct conclusion.

27. A conclusion cannot be drawn without somewhere in the reasoning accepting one or more assumptions.

28. If an argument is logical then the changing of a definition should have no effect on the conclusion of the argument.

29. A false statement may be legitimately used as a reason in a logical argument.

30. If there are three and only three possibilities and one of them must happen, and if two of the possibilities are shown to be in fact impossible, then the third possibility may or may not happen.

31. The statements of an expert should be used as reasons for supporting an argument only in the field in which he is expert.

32. In any argument by elimination of possibilities it is enough to eliminate all but two of the possibilities in order to reach the conclusion.

33. A definition remains unchanged even if the meaning of one of the words in the definition is changed.

34. In a logical argument it is all right to use as a reason a proposition which assumes what is to be proved.

<u>Test IV, Part II.-</u> This test consists of nine geometrical situations. The exercises were chosen well within the experience of the student in an effort to determine to what extent the principles of Part I could be applied to geometrical situations. The situation is usually a discussion or disagreement concerning the correctness of a proof. After a brief description of the circumstances and an exposition of the argument, the student is required to decide whether the argument is logically sound or not, justifying his decision by reference to the pertinent principle or principles of reasoning chosen from a list supplied. This list includes most of the true statements of Part I. A correct choice of supporting principles certainly means, on the part of the student, both comprehension of the principle and also a realization of its application to specific situations.

An "Answer Sheet" is supplied on which the student is to record his decision concerning the situation and his choice of supporting principles.

ANSWER SHEET TEST IV -- Part II

e: ______________________ School: ______________________

: ____(yrs.) ____(months) City or Town: ______________________

de: ________ Sex: ________ Geometry Teacher: ______________

ber of years you have been attending High School (including this year): ________

r standing in class in geometry course you are taking: Check below.
____(lowest 1/3) ____(highest 1/3) ____(middle 1/3)

ber of courses in Demonstrative Geometry you have taken (including courses you are taking this year): ______

Situation I

(1) ______

(2) ______

(3) ______

(4) ______________

Situation II

(5) ______

(6) ______

(7) ______________

Situation III

(8) ______

(9) ______________

Situation IV

(10) ______

(11) ______________

Situation V

(12) ______

(13) ______________

Situation VI

(14) ______

(15) ______________

Situation VII

(16) ______

(17) ______________

Situation VIII

(18) ______________

Situation IX

(19) ______

(20) ______

(21) ______________

STATEMENTS FOR USE IN TEST IV, Part II

The following statements are principles of logical reasoning. You are to use them in support of your decisions in a group of situations to follow.

I. In an argument all the important words should be carefully defined.

II. A definition is of no use unless the people concerned are in agreement about every word used in the definition statement.

III. When the people concerned are agreed about the meaning of a word, a correct conclusion can be reached without a formal definition of the word.

IV. Changing the meaning of one of the words in the statement of a definition has the effect of changing the definition.

V. Different conclusions can be logically arrived at using the same definition if one of the words used in the definition statement is not given the same meaning by all the people concerned.

VI. A word may be correctly defined in several ways.

VII. If one changes a definition the conclusion which follows may also be changed even though the argument is logical.

VIII. The converse of a true theorem is not always true.

IX. In an argument, what is to be proved should not, either directly or indirectly, be used as a reason in the argument.

X. In the proof of a problem no construction should be made which implies or rests upon the statement to be proven.

XI. A conclusion can not be drawn without somewhere in the reasoning accepting one or more assumptions.

XII. In an indirect argument all the possibilities must be considered and all but one must be eliminated.

XIII. To apply a given theorem to a special case, the special case must be shown to satisfy the conditions of the theorem.

TEST IV Part II

ections: You are given an answer sheet and a statement sheet to go
1 this test. In the blanks on the answer sheet you are to answer the
stions asked with regard to the following situations.

Situation I

following are three definitions of ANGLE:-
When two straight lines meet, an angle measured in degrees is formed.
An angle, in degrees, measures the change in direction from one straight
line to another.
An angle, in degrees, measures the amount of rotation of a line about
a point.

1 said:- At quarter after twelve the angle between the hands of a clock is about 90 degrees.

said: - You mean that in the fifteen minutes after twelve o'clock the minute hand of the clock sweeps through an angle of about 90 degrees?

_ Which boy is right? (a) Bill but not John; (b) neither; (c) John but not Bill; (d) both?

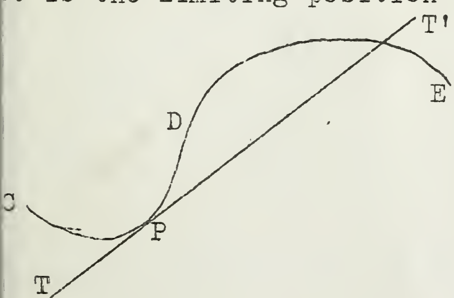
_ Which definition, A, B or C, is John using?

_ Which definition is Bill using?

_ Which of the general statements on your statement sheet apply to this situation? Insert the number of each in blank (4) on your answer sheet.

Situation II

following definitions of TANGENT to a curve were under consideration
1 reference to the figure given below.
It is a straight line which touches a curve at a point.
It is a straight line which however far it is produced has one and only
one point in common with the curve.
It is the limiting position of the secant as the two points in which the secant meets the curve move closer together.



Eunice said:- Definition B. does not apply to this figure since tangent TPT' meets the curve in one point beside the point P.
Nancy replied:- Tangent TPT' meets the curve CD only in one point, hence definition B. applies to the curve CD. DE is another curve.

_ Which girl is right? (a) Eunice but not Nancy; (b) Nancy but not Eunice; (c) neither; (d) both?

_ Over the meaning of what word are the girls in disagreement?

_ Which of the general statements apply to this situation? Insert their numbers in the blank on the answer sheet.

Situation III

e class was discussing the definition of SIMILAR FIGURES as follows:-

ll suggested:- Let us say that two figures are similar when they have the same shape.

nn replied:-- That definition would hardly be satisfactory for the word 'shape' is too vague.

n then offered a definition:- Two figures are similar if their sides are proportional.

ll objected:- The words 'their sides' limit the use of the definition to figures bounded by straight lines. Why not say "Two figures are similar if corresponding lengths are proportional throughout the figures."

nn criticized again:- But have we definitions of 'corresponding lengths' and 'proportional'?

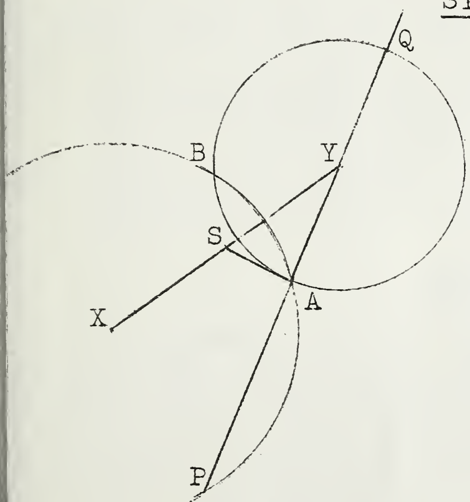
ll defended himself:- We have not formal definitions but from our past work we are able to agree about their meaning, so my definition is workable.

further discussion showed that there was agreement on the meaning of e words in question, would formal definitions of these words be cessary?

8) (a) Yes (b) No

9) Which of the general statements apply to this situation? Insert their numbers in blank (9) on your answer sheet.

Situation IV



X and Y are the centres of two circles. The circles intersect at A and B.

XS = SY

SA is tangent to the circle BAQ.

PQ is perpendicular to SA.

With these data Dick proceeded with the following argument:-

By one of our theorems we know that the perpendicular from the centre of a circle to a chord bisects the chord. Take S as the centre of a circle, with PQ as a chord.

Then AP = AQ.

e class found that the theorem referred to had been correctly stated.

10) Would you accept the above proof as satisfactory? (a) Yes (b) No.

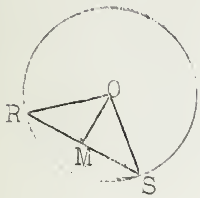
11) List on your answer sheet the number of each general statement you would use to support your conclusion.

Situation V

lass was trying to prove that the line joining the midpoint of a rd RS to the centre O of a circle is perpendicular to the chord.

John offered the following proof:-

OR and OS are radii of the circle. Hence OR=OS. Therefore triangle ORS is isosceles. There is a theorem which states that the perpendicular bisector of the base of an isosceles triangle passes through the vertex. Therefore MO is perpendicular to RS.



The class agreed that John had correctly stated the theorem in question, but some members of the class thought that his proof was not logical.

12) Do you think that John's proof is logical? (a) Yes (b) No.

13) Which of the general statements explain your decision. Insert their numbers in the blank on the answer sheet.

Situation VI

tuation V was still under consideration. Mary claimed that John's oof was not satisfactory, but that she had one which was logical. e stated it as follows:-

We know that the perpendicular from the centre of a circle to a ord bisects the chord. But M is the midpoint of the chord RS, therere MO must be perpendicular to RS.

e class agreed that the underlined statement was correct.

14) Accept their verdict. Do you agree that Mary has proved that the line joining the midpoint of a chord to the centre of a circle is perpendicular to the chord? (a) Yes (b) No.

15) Which of the general statements would you use to support your decision? Insert the number of each in the blank (15) on the answer sheet.

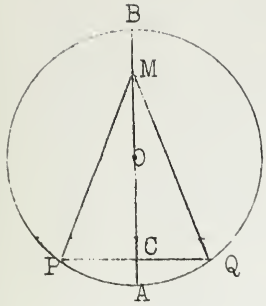
Situation VII

a class discussion of the parallelogram a model made from laths, lted together at the ends, was used. Various properties of the model re noted and it was finally decided to define the parallelogram as a ur-sided figure whose opposite sides are parallel. On watching the agonals of the model for various positions Bill remarked that in any rallelogram the diagonals bisect each other. The class was able to ove that Bill's statement was correct. Jack then said "I've been etching quadrilaterals on paper and it is quite clear to me that we ve proved that if the diagonals of a four-sided figure bisect each her, then the figure is a parallelogram.

16) Do you believe that the class had proved the underlined statement? (a) Yes (b) No.

17) Insert the number of each reason you feel supports your conclusion.

<u>Situation VIII</u>



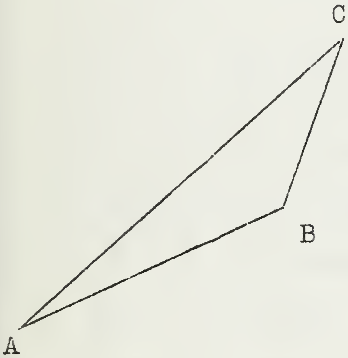
AB is a diameter of a circle, centre O.
M is any point between A and B on AB.
Construct PM and QM such that
$\angle PMA = \angle QMA$.

Betty argued as follows to prove that
PM = QM: --

Construct PC and QC perpendicular to AB.
On comparing triangles CMP and CMQ,
CM is common, $\angle CMP = \angle CMQ$, etc.

Betty's argument is already in error.

18) Insert on the answer sheet the number of each of the general statements which tells why the above is in error.



<u>Situation IX</u>

A group of students were to prove that "If in the triangle ABC angle B is greater than angle A, then the sides BC and AC are unequal and AC is greater than BC."

Ralph offered the following proof.

If AC = BC
then $\angle A = \angle B$
but we are told that $\angle A$ is not equal to $\angle B$.
Therefore AC and BC can not possibly be equal and must be unequal.
Hence AC is greater than BC, since it is not equal to BC.

19) Did Ralph prove that AC and BC are unequal?
(a) Yes. (b) No.

20) Did he prove that AC is greater than BC?
(a) Yes. (b) No.

21) Write the number of each general statement which supports your decision concerning Ralph's proof that AC is greater than BC.

Test V.- A fifth test, concerned with the ability of pupils to reason in non-geometric situations, was prepared by S.C.Clarke, M.A. A copy of Test V is attached to this report for completeness.[1] Mr. Clarke has described the test and his findings in a separate report.

The Distribution of the Test Battery.

The test Battery has been distributed among six provinces, Nova Scotia, Quebec, Ontario, Alberta, and British Columbia, the greatest number being kept in Alberta. The results examined here are based on only the Alberta returns.

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1. Appendix 1.

CHAPTER III

THE MARKING OF THE TESTS AND THE TABULATING OF RESULTS.

Original Marking and Recording of Student Answers.

Tests I, II and Test IV, Parts I and II were objective in form and were quite easily marked. The results were first tabulated on large sheets with the question numbers written across the top and the students' names, grouped in schools, down the left hand margin. This gave a two way table, such that horizontal addition gave for each pupil the number right and the number wrong on each individual question. Adding these last totals gave the total results on each of the questions. The size of these record sheets makes their inclusion in this report impossible. Nevertheless they were useful in supplying detailed information, the obtaining of which would otherwise have necessitated the re-examination of the original answer papers.

Tabulating the Results of the Individual Tests.

Test I.- This test has sixteen blanks to be filled by the students. On the large record sheets each answer was recorded as wrong or correct by an x or by a Check mark. The pupils being grouped by schools on the record sheet, it was possible to find totals for each school. The total number of correct responses (sum of check marks) and the total number of attempts at a given question (sum of checks and x's) were found. The ratio of these two numbers, in each case, was converted to a percentage. These results by school and by question are given in Table I. For example, 22 pupils in School I tried question I, 17 got it correct, giving a per cent score of 77. The total questions tried for each school, the total questions correct for each school and per cent score of the school on Test I are given at the bottom of Table I. Thus School VIII had a cumulative correct score of 492 out of 577 questions tried giving a per cent score of 85. This last figure provides a basis on which to compare the achievement of the various schools for Test I.

The test Battery was given to eleven schools in Alberta where the students were currently studying Geometry. Each school is referred to by a number, the derivation of which is explained in Chapter IV.

TABLE I
RESULTS BY SCHOOLS ON TEST I

Question	School I	II	III	IV	V	VI
I	17 22 77	6 7 86	7 8 88	14 17 82	6 6 100	7 7 100
II	21 22 95	6 7 86	9 10 90	18 18 100	6 6 100	7 7 100
III	21 22 95	6 6 100	9 9 100	18 18 100	5 6 83	6 7 86
IV	10 19 53	3 3 100	6 8 75	12 18 67	4 6 67	5 6 83
V	17 19 90	4 4 100	7 8 88	13 17 77	5 5 100	7 7 100
VI	20 21 95	4 4 100	7 10 70	18 18 100	5 6 83	7 7 100
VII	18 22 82	6 7 86	5 8 63	15 18 83	6 6 100	4 7 57
VIII	15 22 68	6 7 86	9 10 90	16 18 89	5 6 83	6 7 86
IX	13 20 65	6 7 86	6 10 60	13 18 72	6 6 100	6 7 86
X	15 21 71	6 7 86	8 10 80	14 18 78	6 6 100	6 7 86
XI	16 22 73	7 7 100	8 10 80	16 17 94	6 6 100	6 7 86
XII	17 22 77	6 7 86	7 10 70	15 17 88	6 6 100	5 7 71
XIII	22 22 100	6 7 86	9 9 100	18 18 100	6 6 100	7 7 100
XIV	15 21 71	5 6 83	8 8 100	14 17 82	5 6 83	7 7 100
XV	14 17 82	5 6 83	9 9 100	13 15 87	4 5 80	7 7 100
XVI	14 22 64	4 6 67	7 10 70	13 18 72	6 6 100	4 7 57
Total	265 336 79	86 98 88	121 147 82	240 280 86	87 94 93	97 111 87

TABLE I (CONT'D)

Question	School VII			VIII			IX			X			XI		
I	15	17	88	28	36	78	25	35	71	5	14	31	14	26	54
II	17	18	94	33	37	89	32	35	91	14	14	100	29	32	91
III	17	19	90	31	37	84	26	30	87	10	16	63	15	30	50
IV	10	16	63	24	36	67	5	26	19	5	9	56	8	19	42
V	13	16	81	22	33	67	23	28	82	7	9	78	24	25	96
VI	15	16	94	35	37	95	29	31	94	11	12	92	27	30	90
VII	14	17	82	31	36	86	21	31	68	7	14	50	18	31	58
VIII	18	18	100	35	37	95	29	31	83	12	14	86	26	30	87
IX	14	18	78	28	37	76	23	35	72	11	14	79	19	27	70
X	14	18	78	33	37	89	24	32	75	7	14	50	18	26	69
XI	14	18	78	35	37	95	26	32	81	10	14	71	21	29	72
XII	15	17	88	33	37	89	28	33	85	11	14	79	20	26	77
XIII	16	17	94	37	37	100	29	30	97	11	12	92	23	23	100
XIV	13	15	87	29	33	88	22	28	79	7	11	64	5	16	31
XV	14	16	88	31	33	94	25	27	93	9	11	82	9	9	100
XVI	17	19	89	27	37	73	22	31	71	9	14	64	9	15	60
Total	236	275	85	492	577	85	389	496	78	146	206	71	285	394	70

Test II.- Test II has a total of 79 blanks to be filled in as responses in the solution of six exercises. The classification of the responses within each exercise as "Answers", "Statements" and "Reasons" along the line indicated in the description of the test in Chapter II,[1] gave a total of fifteen scores on Test II. For example, an examination of Exercise I shows that blank 6 is an "Answer", blanks 1, 2 and 3 are "Statements" and blanks 4, 5 and 7 are the "Reasons" for these statements. The fifteen groups are indicated in the first column of Table II, Page 43. The total number of correct responses from a school on each of these fifteen groups was converted to a percent of the possible number of correct responses. To derive a School's score on the entire test the total number of correct answers from the school was converted to a per cent of the total number possible. This percentage, which is indicated in the bottom row of Table II, is comparable for schools, whether they are large or small.

TABLE II
RESULTS BY SCHOOLS ON TEST II

Blanks and Classification	School I	II	III	IV	V
I A-6	14 22 64	6 7 86	5 10 50	16 17 94	4 6 67
S-1,2,3	63 66 96	19 21 90	26 30 87	51 51 100	17 18 95
R-4,5,7	51 66 77	13 21 62	16 30 53	48 51 94	12 18 67
II A-18	21 22 95	7 7 100	10 10 100	17 17 100	6 6 100
S-8,9,10,12, 14,16,17	142 154 92	48 49 98	65 70 93	71 119 60	38 42 91
R-11,13,15	59 66 89	15 21 71	24 30 80	43 51 84	13 18 72
III S-20,22,23 24,25,26	108 132 82	39 42 93	44 60 73	72 102 70	32 36 89
R- 19,21,27	56 66 85	21 21 100	24 30 80	50 51 98	18 18 100
IV S-29,30,32,33 35,36,37,38, 39,40	168 220 76	60 70 86	73 100 73	94 170 55	45 60 75
R-28,31 34,41	80 88 91	26 28 93	37 40 93	62 68 91	24 24 100
V A-59,63	26 44 59	7 14 50	12 20 60	23 34 68	9 12 75
S-42,43,44,45,46 47,48,49,52,53 54,55,56,58,61	287 350 82	81 105 77	118 150 79	200 255 78	79 90 88
R-50,51,57, 60,62,64	109 132 83	37 42 88	38 60 63	64 102 63	25 36 89
VIS-66,67,68,70 72,73,74,75, 77,78	192 210 91	56 70 80	77 100 77	128 170 75	57 60 95
R-65,69,71, 76,79	64 105 61	20 35 57	21 50 42	43 85 51	24 30 80
	1542 1743 89	455 553 82	590 790 75	982 1363 72	393 474 83

TABLE II (CONT'D)
RESULTS BY SCHOOLS ON TEST II

	School					
	VI	VII	VIII	IX	X	XI
I	5	19	29	17	5	18
	7	20	37	31	16	33
	71	95	78	55	31	55
	19	60	104	88	44	92
	21	60	111	93	48	99
	90	100	94	95	92	93
	16	57	85	57	29	81
	21	60	111	93	48	99
	75	95	77	61	60	82
II	7	20	36	27	14	32
	7	20	37	31	16	33
	100	100	97	87	88	97
	45	134	244	199	90	195
	49	140	259	212	112	231
	92	96	94	94	80	85
	15	49	98	74	36	77
	21	60	111	93	48	99
	71	82	88	80	75	78
III	27	95	164	112	53	119
	42	120	222	186	96	198
	64	79	74	60	55	60
	16	52	97	70	34	80
	21	60	111	93	48	99
	76	87	87	75	71	81
IV	51	137	280	192	93	193
	70	200	370	310	160	330
	73	68	76	62	58	58
	24	72	126	93	53	109
	28	80	148	120	64	132
	86	90	85	78	83	83
V	10	24	46	31	20	31
	14	40	74	56	32	62
	72	60	62	55	63	50
	88	241	476	346	187	333
	105	285	555	450	240	480
	59	85	86	77	78	69
	28	83	143	104	56	112
	42	114	222	180	96	192
	67	78	64	58	58	58
VI	60	154	308	213	96	223
	70	190	370	290	140	310
	86	81	83	73	69	72
	22	60	144	72	41	82
	35	95	185	145	70	155
	63	63	78	50	59	53
	433	1257	2380	1695	851	1777
	553	1544	2923	2383	1234	2552
	78	81	81	71	69	70

Test III.- Test III is not a completion test. The intention was to examine the answer papers for only certain schools which the other tests indicated to be either typical or exceptional. The results from the study of Test III are related in a later Chapter.[1]

Test IV, Part I.- As was indicated in Chapter II, the statements which the student was to designate as True or False were arranged in pairs, one true, one false, yet both based on the same principle.

Of the seventeen pairs, ten require a comprehension of the following ideas:

(a) the importance of definition,

(b) the relation between definition and conclusion,

(c) the relation between assumptions and conclusions,

(d) the technique of indirect argument.

The results for these principles are given in Table III.

The seven remaining pairs, recorded in Table IV, are based on the following common fallacies in reasoning:

(a) personal attack

(b) assuming a converse

(c) the use of false authority

(d) the use of irrelevant statements, and

(e) arguing in a circle.

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1. Chapter V.

A correct response for both members of the matched pair of statements was recorded as "Correct". If the statements were shown both true or both false, the combined reply was recorded as a "Guess". The inconsistency would appear to indicate the student did not understand the statements. An incorrect response for each statement was recorded as "Wrong". In Tables III and IV the total number of pupils in each school with the rating "Correct", "Wrong" or "Guess", is recorded for each pair of statements. To give the school its score the total "Correct" was expressed as a per cent of the number that school could possibly get correct (the product of the number of pupils and the number of pairs).

TABLE III

RESULTS BY SCHOOLS ON TEST IV PART I - PRINCIPLES

School		1 & 21	5 & 33	8 & 16	2 & 24	9 & 23	20 & 28	3 & 32	18 & 30	14 & 26	15 & 27	Total
I	C	21	21	15	17	8	19	12	17	19	4	153
	W	0	0	5	0	7	0	1	1	0	5	220
	G	1	1	2	4	7	3	9	4	3	12	70
II	C	7	6	5	5	1	6	6	7	5	0	48
	W	0	1	0	1	2	0	0	0	0	0	70
	G	0	0	2	1	4	1	1	0	2	7	69
III	C	10	9	3	7	5	7	8	9	8	0	66
	W	0	0	4	0	5	0	1	1	0	5	100
	G	0	1	3	3	0	3	1	0	1	4	66
IV	C	16	18	11	13	2	15	14	12	10	3	114
	W	2	0	2	0	10	0	0	0	0	4	190
	G	1	1	5	5	3	2	1	2	6	12	60
V	C	5	5	3	5	2	5	5	3	5	1	39
	W	0	1	3	0	3	0	0	2	0	1	60
	G	1	0	0	1	1	1	1	1	1	4	65
VI	C	6	5	2	6	1	4	4	5	4	2	39
	W	0	1	1	0	4	0	1	1	2	3	70
	G	1	1	4	1	2	3	2	1	1	2	56
VII	C	16	13	7	7	4	12	11	13	14	2	99
	W	0	2	3	1	4	0	1	1	0	5	170
	G	1	2	7	9	9	5	5	3	3	9	58
VIII	C	33	30	16	24	6	15	19	21	27	5	186
	W	0	2	8	1	8	4	1	8	3	9	370
	G	4	4	12	11	13	17	15	8	7	19	50
IX	C	28	27	9	17	9	21	22	21	23	7	184
	W	0	1	7	2	19	2	2	1	1	10	340
	G	6	6	18	15	6	9	10	9	10	14	54
X	C	13	15	6	11	4	13	13	15	12	4	106
	W	0	0	2	0	9	0	0	0	0	2	160
	G	3	1	8	5	2	3	3	1	4	10	66
XI	C	26	29	11	16	8	19	24	21	19	7	180
	W	2	3	5	4	17	1	1	4	0	6	330
	G	5	1	17	13	8	11	8	8	14	18	55

TABLE IV

RESULTS BY SCHOOLS ON TEST IV PART I-FALLACIES

School		4 & 13	11 & 22	6 & 17	7 & 29	10 & 31	12 & 19	25 & 34	Total
I	C	16	16	18	20	16	9	6	101
	W	0	1	3	0	0	0	4	154
	G	5	5	1	2	6	13	2	66
II	C	4	5	6	5	7	3	3	33
	W	0	0	0	0	0	0	1	49
	G	3	2	1	2	0	4	3	67
III	C	10	8	7	8	9	6	9	57
	W	0	0	0	0	0	0	0	70
	G	0	2	3	2	1	4	1	81
IV	C	14	9	9	9	16	10	4	71
	W	0	0	8	0	0	0	9	105
	G	1	6	1	7	3	9	3	67
V	C	4	4	3	4	4	2	0	21
	W	0	1	1	0	0	0	1	42
	G	2	1	2	2	2	4	5	50
VI	C	7	5	2	5	6	6	3	34
	W	0	0	4	0	0	1	1	49
	G	0	2	1	2	1	0	3	69
VII	C	13	10	6	16	13	8	6	72
	W	1	3	10	0	0	2	5	119
	G	3	4	1	1	4	7	6	60
VIII	C	20	19	24	30	23	19	21	156
	W	2	4	6	0	4	2	5	259
	G	15	14	6	4	10	16	11	60
IX	C	21	12	16	24	22	13	18	126
	W	0	7	14	0	2	4	6	238
	G	11	15	4	9	10	16	10	53
X	C	11	7	8	11	9	5	9	60
	W	2	1	5	0	5	2	2	112
	G	3	7	3	5	2	9	5	54
XI	C	22	16	17	22	21	8	8	114
	W	1	4	11	0	1	4	14	231
	G	10	11	5	9	11	21	11	49

Test IV, Part II.- In Test IV Part II the student was required to select a correct conclusion from a multiple choice arrangement. Then, from a list of principles of logic provided, the student was required to select those principles supporting the conclusion chosen. Since selecting a correct conclusion would seem to be a different kind of action from spotting supporting reasons, the two processes are at first handled separately. In the conclusion bracket of Table V is given the number of conclusions correct, followed by the possible number (number of pupils times the number of conclusions to be chosen in the test - 12) and the resultant per cent score. In the next section of the table the same procedure is followed with regard to correct reasons selected. Handling irrelevant reasons necessitated a slightly different procedure. Since the best possible that could be expected in this regard would be to choose no irrelevant reasons, the number of such reasons chosen can not be immediately converted to a per cent score. Instead the number of irrelevant reasons was subtracted from the number of correct reasons; the difference was then converted to a percentage of the possible number of correct reasons (column 2 in reasons bracket).

To get a single score for the entire Part, reasons and conclusions were combined. The number of correct conclusions was added to the number of correct reasons and from this total was subtracted the number of irrelevant reasons the student offered to support his conclusions. By stating the result as a per cent of the possible total, the score shown in the last column of Table V was secured for each school.

RESULTS BY SCHOOL ON TEST IV PART II

School	Conclusions			Reasons						Combined		
	Corr	Poss	%	Corr	Poss	%	Irrel	Corr Irrel	%	CC+RC -IR	Poss	%
I	162	252	64	202	483	42	113	89	18	251	735	34
II	61	108	56	79	207	38	58	21	10	82	315	26
III	74	120	61	82	230	36	45	37	16	111	350	32
IV	144	228	63	152	437	35	72	80	18	224	665	34
V	30	60	50	40	115	35	54	-14	-12	16	175	9
VI	45	84	54	70	161	43	66	4	2	49	245	20
VII	121	216	56	134	414	32	149	-15	4	106	630	17
VIII	Test IV Part II missing											
IX	193	384	50	187	736	25	173	14	2	207	1120	18
X	89	180	50	111	345	32	115	-4	-1	85	525	16
XI	173	396	44	203	495	41	158	45	9	218	1155	19

Corr.----Correct
Poss.----Possible
Irrel.---Irrelevant
C.C.+R.C. - I.R.----Conclusions correct plus reasons correct minus irrelevant reasons.

Deriving Basic Table.

From the preceeding work, for each school there are five per cent scores, one on each of the following: Test I, Test II, Test IV, Part Ia and Test IV Part Ib and Test IV Part II. Although all these scores are per cents, they do not seem to offer a satisfactory basis for comparing achievement of the respective tests since the ranges of the scores vary widely with the test. For example, the per cent scores of Test I are high, while those of Test IV Part II are low. To get these scores in a form where they are comparable, the results of the schools on each test were distributed from 0 to 10; 0 going to the lowest score (70% for Test I) and 10 going to the highest score (93% for Test I). Intermediate scores were scaled linearly. Thus a score of 85 is transformed to $\frac{(85 - 70)}{(93 - 70)} \times 10 = 6.5$ on the new scale. Consequently, with each per cent score converted to an "Index" between 0 and 10, each school has five "Index" scores to indicate its achievement relative to other schools. The indices offer, also, a means of comparing the attainment of a given school on the various tests. In Table VI the per cent scores (taken from the table indicated at the head of the column) are recorded with the corresponding "Index" scores. Inasmuch as this table is the basis on which a comparison of schools is attempted it will henceforth be referred to as the "Basic Table".

[illegible]

[illegible]

TABLE VI

BASIC TABLE

SHOWING INDEX AND PER CENT SCORES

Results of	Test I		Test II		Test IV Part Ia		Test IV Part Ib		Test IV Part II	
Source	Table I		Table II		Table III		Table IV		Table V	
School	%	Index	%	Index	%	Index	%	Index	%	Index
I	79	3.9	88	10	70	10	66	5.3	34	10
II	88	7.8	82	6.5	69	9.5	67	5.6	26	6.8
III	82	5.2	75	3.0	66	8.0	81	10	32	9.2
IV	86	7.0	72	1.5	60	5.0	67	5.6	34	10
V	93	10	83	7.0	65	7.5	50	0.3	9	0
VI	87	7.4	78	4.5	56	3.0	69	6.3	20	4.4
VII	85	6.5	81	6.0	58	4.0	60	3.4	17	3.2
VIII	85	6.5	81	6.0	50	0	60	3.4	--	--
IX	78	3.5	71	1.0	54	2.0	53	1.3	18	3.6
X	71	0.4	69	0	66	8.0	54	1.6	16	2.8
XI	70	0	70	0.5	55	2.5	49	0	19	4.0

CHAPTER IV

RATING OF THE SCHOOLS AND OF THE TESTS.

Charts I & II - Graphical Comparison of Schools.

With the results of the schools on each test reduced to an index, an attempt was next made to rate the school. Possibly the rating of schools needs a certain amount of justification. With over 200 students each filling in over 200 blanks, it was impossible to give to all schools the detailed examination which, in a study of this type, gives the most enlightening and useful results. If the schools could be classified, then one school from each class might be chosen as representative of that group. The conclusions obtained from a detailed examination of the one school it was hoped would yield results that could be taken as representative. The basis of classification or rating of schools was not easily selected. Charts I & II were obtained by arranging the tests along the vertical scale and plotting the results from each school.[1]

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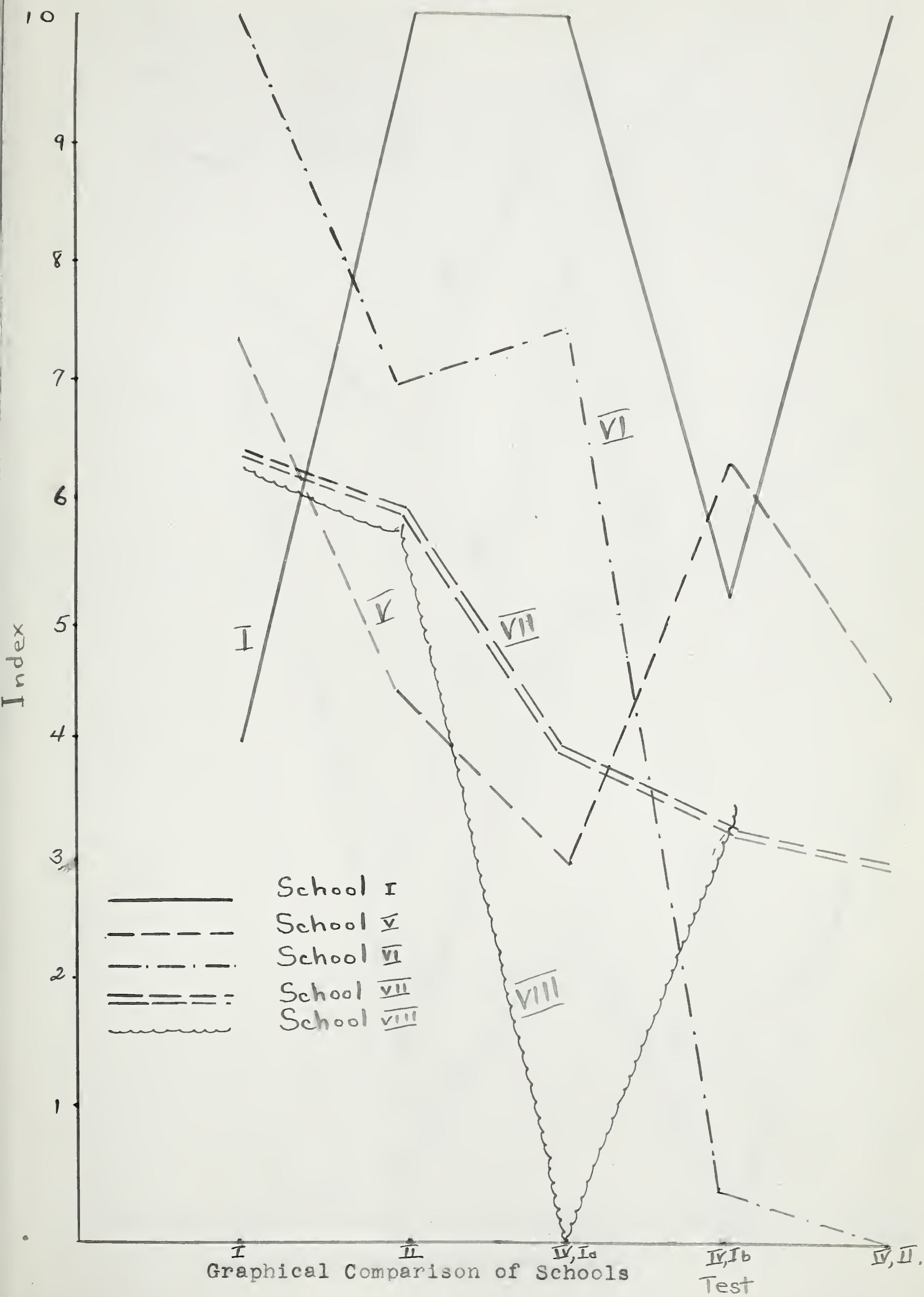
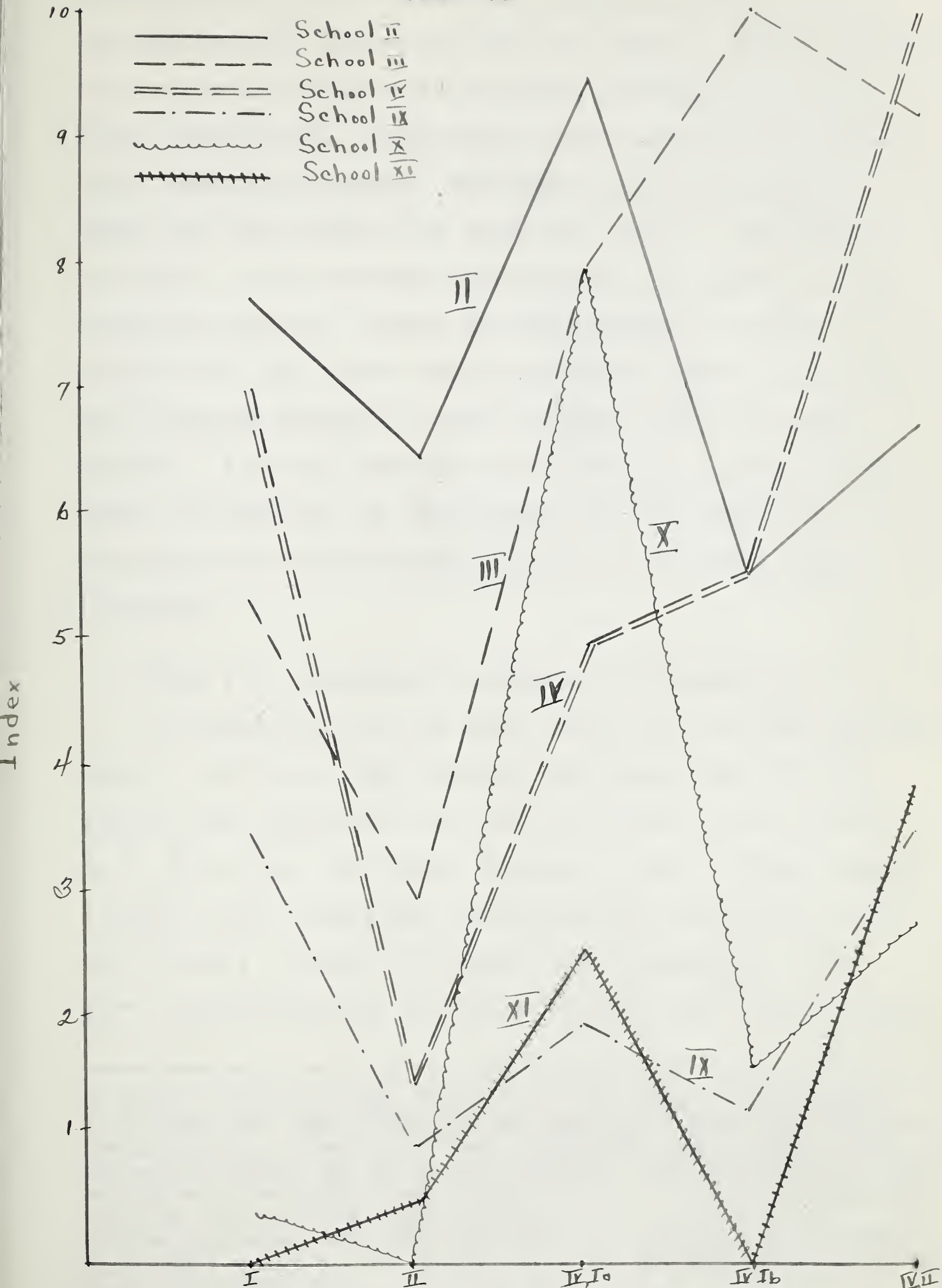
1. The plotting of all eleven schools on one chart as was first attempted gave so many lines that the chart was useless to determine even trends. Consequently two charts were used with the Category A and Category C schools on Chart I and Category B and D schools on Chart II. The terms A,B,C and D will be explained later.

CHART I



Graphical Comparison of Schools

CHART II



Graphical Comparison of Schools

Examination of this chart reveals that no schools are uniformly above all others or below all others. Indeed it would be surprising if such were the case. Instead we find the lines intersecting, indicating a school may do well on one test, poorly on another. Yet there are certain schools which, on the average, lie above the others. For example on Chart I it is apparent that School I is superior to the remaining schools. School VI seems unusual. On Chart II Schools II, III, IV are definitely above Schools X, XI and XII, although School X's score on Test IV Part Ia seems unusual. All this indicates that there is a certain hierarchy of standing, but these charts are not sufficient to determine the relative positions of the schools in this hierarchy.

Table VII - Grouping the Schools in Categories.

In basic Table VI the index scores are recorded for each school. To try another approach the index scores of the schools were grouped as follows: 8-10 as A; 5-8 as B; 3-5 as C; 0-3 as D.[1] The number of times a given school attained a given letter rating was counted and the school placed in the category in which it occured most frequently. This gave a distribution as indicated in Table VII.[2] The fraction

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1. These letter ratings A,B,C,D may be thought of as High, Good, Fair and Low, although perhaps these words have undesirable connotations. After all "low" only means low in this test, and with respect to the other schools. Hereafter the letter rating will be used; the reader may if he desires substitute the more descriptive word rating.
2. The numbering that has been used to identify the schools up to this point in the study has been the rating given to the school in Table VII.

following the school, indicates the number of times, out of five, the school attained the class in which it is placed or better. Of the eleven schools from which records are available, one fell in Category A, three fell in Category B, four in Category C and three in Category D.

TABLE VII

THE SCHOOLS GROUPED IN CATEGORIES.

Category A Index 8-10	Category B Index 5-8	Category C Index 3-5	Category D Index 0-3
School I 3/5	School II 5/5	School V 3/5	School IX 3/5
	School III 4/5	School VI 3/5	School X 4/5
	School IV 4/5	School VII 3/5	School XI 4/5
		School VIII 3/5 (Incomplete)	

Chart III - A check on the Rating of Schools as given in Table III.

Chart III was used to study further the soundness of the rating given in Table VII with respect to Basic Table VI, and also to determine the relative difficulty of the tests. Placing the schools along the horizontal scale in the order of their rating and plotting percentage scores, gave Chart III. Each broken line represents the results of the various schools on each one of the tests. Vertical lines are inserted to classify the schools according to the rating given in Table VII.

If the rating given in Table VII is sound then the schools to the left on Chart III should make higher scores than those to the right. In other words a negative slope would seem to confirm the adequacy of the rating. Let us look at each of these broken line graphs. In both Tests I and II, while there is no significant difference between B and C Schools, still the D group of Schools scored consistently lower than the other groups. C schools are below schools rated as A and B in both sections of Test IV Part I, although in these tests D Schools are almost equivalent to C schools. The broken line for Test IV Part II clearly has a downward trend as it progresses to the right.

In summary it might be said that Chart III indicates the rating in Table VII to be fairly reliable. At least Schools I-IV are definitely superior to Schools IX-XI and Schools V-VIII[1] seem to be an intermediate group. However no test gives a broken line graph with a negative slope throughout. Thus no single test would give the same rating as that secured by combining the five scores as in Table VII. Still the general trend of all the broken lines indicates the rating to be sound.

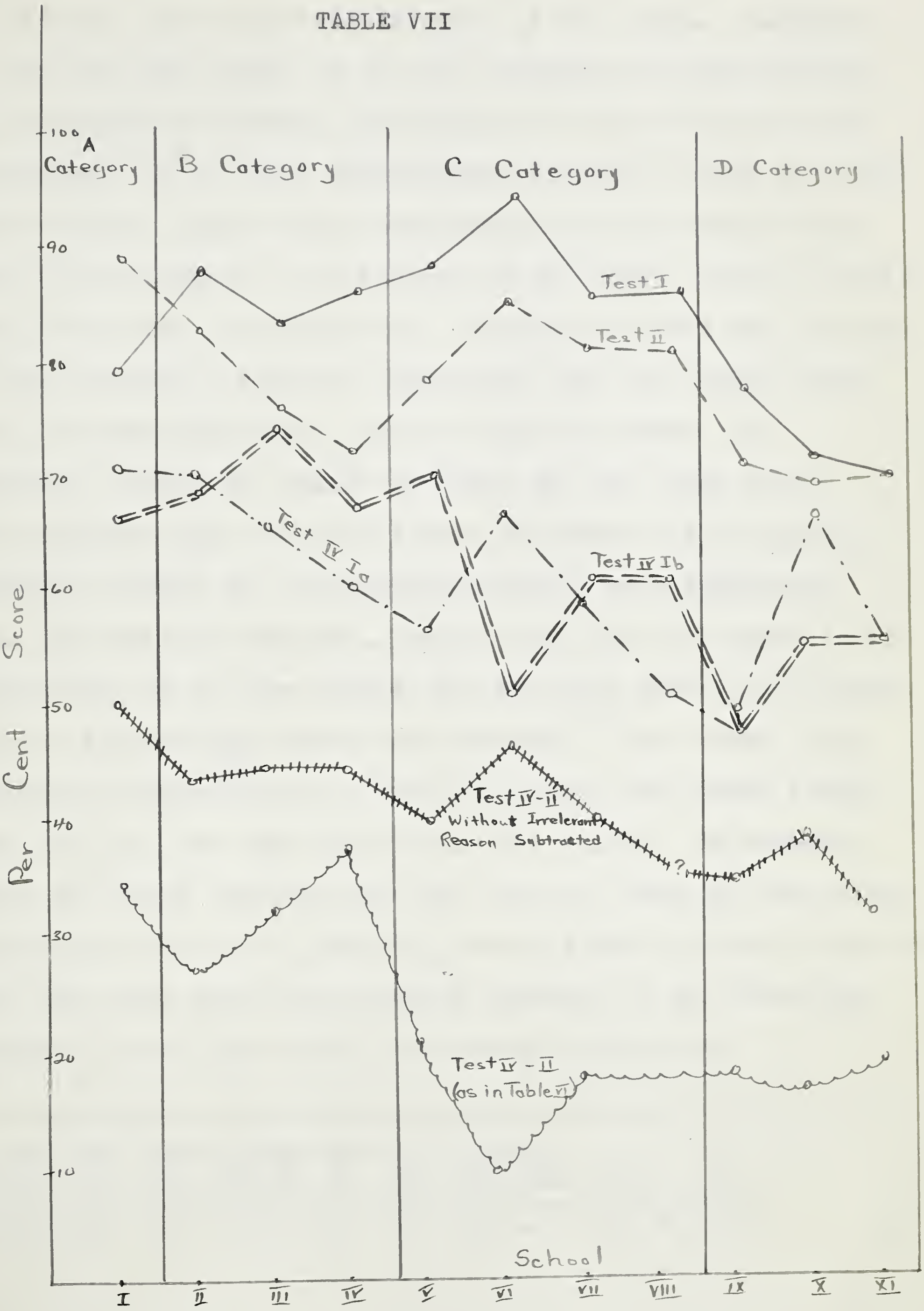
---

1. Note that School VI would be in Category B if reasons correct score had been used without irrelevant reasons being subtracted.

CHART III

A CHECK ON THE RATING OF THE SCHOOLS AS GIVEN IN

TABLE VII



Rating of Tests by Chart III.

The second use that was to be made of Chart III, was to indicate the relative difficulty of the tests. Inasmuch as the per cent scores on the test depends so predominantly on the method of marking, the degree of difficulty will not necessarily be of great significance although it may be quite interesting. Lying almost completely above the other tests Test I would appear to be easiest of the group. Test II would seem to be next in difficulty. To choose between the sections of Test IV Part I would be impossible, for one school found one part more difficult, another found the other. In evidence, notice the number of times the two lines cross. The scores on Test IV Part II were the lowest of the group, probably because the "Irrelevant Reasons" were subtracted from the "Correct Reasons". Suspecting that this might be an explanation of the low scores, the per cent scores were found without subtracting "Irrelevant Reasons".[1] The broken line obtained on Chart III as a result is above the former broken line for this test but still below the others. In summary, Chart III would indicate that the tests in order of increasing difficulty were Test I, Test II, Test IV Part I, Test IV Part II. Does this mean that the increased emphasis on the reasoning process is the cause of the increasing difficulty?

---

1. The per cent scores are as follows:

School	I	II	III	IV	V	VI	VII	VIII	IX	X	XI
Per cent	50	44	45	45	40	47	40	--	34	38	32

Chart IV and Table VIII

A study of Improvement of Schools as Tests Proceeded.

With the schools rated as to attainment and the tests as to difficulty, the next question to ask is, "Did any schools improve as the tests proceeded?" Now it must be noted that improvement is relative, one school to another. The index for a school on the second test is improved if that school did better on the second test, relative to the other schools, than it did on the first test. This limitation on the meaning of improvement must be kept in mind. If a school improved, that improvement is indicated by an increase in the index number (Table VI) or by a positive slope of the line segment in Charts I and II. These results are indicated in Table VIII. Since there are five tests there are four line segments. Column three of Table VIII indicates the number of these which are positive or negative. Column Four records the mean slope of these line segments calculated by dividing the sum of the slopes by four. Chart IV illustrates these results with the schools distributed along the horizontal axis and the mean slope along the vertical axis.

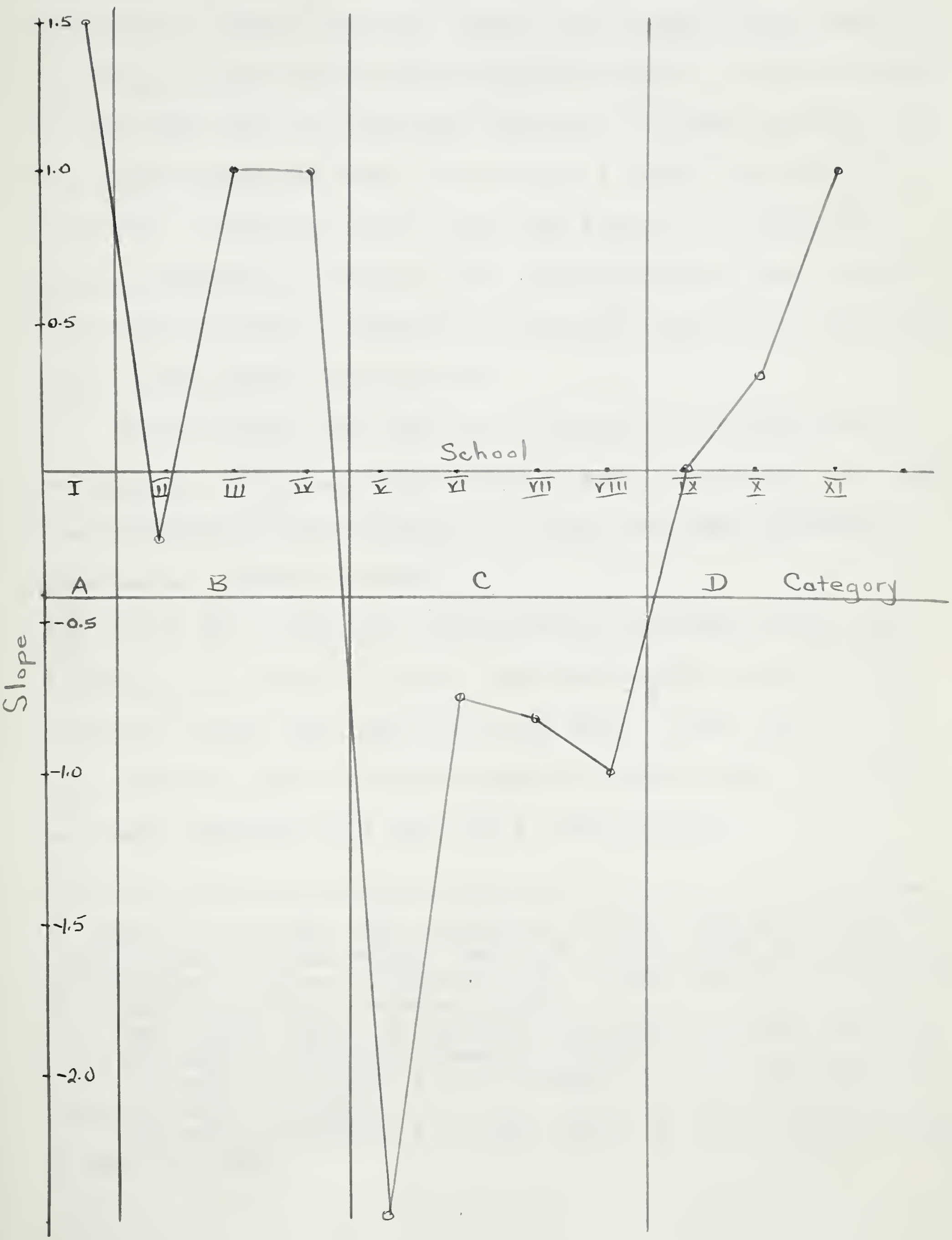
TABLE VIII

IMPROVEMENT OF SCHOOLS

Category of School	Rating of School	No. of Segments (Chart I or II) of sign indicated		Mean Slope
		Negative	Positive	
A	I	1	2	1.5
	II	2	2	--0.25
B	III	2	2	1.0
	IV	1	3	1.0
	V	3	1	-2.5
C	VI	3	1	-0.75
	VII	4	0	-0.8
	VIII	2/3	Incomplete	-1
	IX	2	2	0
D	X	2	2	0.6
	XI	1	3	1.0

CHART IV

IMPROVEMENT OF SCHOOLS ON TESTS



Since Schools in Categories A and B tend to be above the horizontal axis on Chart IV it is evident that these schools improved as the tests progressed, in the matter of position relative to other schools.[1] Thus, for example, while School III falls in per cent score from 82 on Test I to 32 on Test IV Part II, still it improves relative to other schools, for its index increases from 5.2 in Test I to 9.2 in Test IV Part II. A negative mean slope for schools of the middle group (Category C) implies that they tended to lose place, while the schools of Category D changed very little, the mean slope being small but positive.

If we recall that the tests become successively more reliant on reasoning, more abstract and theoretical, and less the mechanical "get-the-answer" type, then the following conclusions become evident:

(1) The A and B schools, while showing no superiority over C schools in the early tests, are noticeably better as the problems stress the logic more and more.[2] (Will Test V show schools I-IV with still greater superiority, i.e. can we teach geometry well and still teach logic?)

---

1. School II is the only exception and its negative slope is small (0.25) so the words "tend to" do not seem too strong.
2. The Index scores of Schools I to IV on Test I are 3.9, 7.8, 5.2 and 7.0 giving a mean of 5.98.
   The index scores of Schools V to VIII on Test I are 10, 7.4, 6.5 and 6.5 giving a mean of 7.60.
   On Test II Schools I to IV score 10, 6.5, 3.0 and 1.5 giving a mean of 5.25.
   On Test II Schools V to VIII score 7, 4.5, 6 and 6 giving a mean of 5.88.

(2) Although Schools of Category C were superior to Category A and B Schools in the early tests they lost ground in later tests. Considering the nature of the tests this would apparently indicate that in these schools students have learned geometry in the old sense but are not cognizant of the concepts of logical thinking that underlie their work.

(3) The Category D schools are inferior in all tests. Still the positive mean slopes on Chart IV would imply that the schools improved as the tests proceeded--or in other words the inferiority decreases.[1] This makes it dubious whether we can conclude that inferiority in "getting answers" is due to lack of ability in reasoning techniques. Probably the two deficiencies are concomitant consequences of some other factors.

The fact that the foregoing general statements can be made concerning the various categories would indicate the possibility of choosing one school from each category as representative of that category.

---

1. This at first seems contradictory to the trend of tests in Chart III where the general slope seems about the same for all tests. By way of explanation it might be pointed out that Chart III is in terms of per cent scores, which, as it was pointed out earlier, are not comparative. Thus an improvement of 10% on Test I is not of the same significance as an improvement of 10% on Test IV, yet the graphs of Chart III would have the same slopes.

CHAPTER V

COMPARING TYPICAL SCHOOLS

Choice of Typical Schools

As typical of the four categories in which the schools were placed, Schools I, IV, VII and X were chosen for detailed study. Within each category the school with the largest number of pupils was chosen in order to have the samples as large as possible. The number of geometry pupils for which the records are complete or nearly so varied from 15 to 21 in the schools chosen.

At this point it might be well to recall the purpose of the detailed and comparative study of the individual schools. We already know with regard to our typical schools that, on the whole, School I excelled, while School IV performed better than Schools VII and X. An attempt is now to be made to determine the cause of the superiority. Various factors that might be thought to be responsible are now considered.

Mental Age of the Typical Schools.

Table IX shows the distribution of mental ages for the typical schools.

TABLE IX

DISTRIBUTION OF MENTAL AGES OF PUPILS IN FOUR TYPICAL SCHOOLS

Mental Age	Category D School X	Category C School VII	Category B School IV	Category A School I
$13^{0}-13^{5}$	60,61 [#]		225	
$13^{6}-13^{11}$				
$14^{0}-14^{5}$	49		224	
$14^{6}-14^{11}$		21,25	222,223	
$15^{0}-15^{5}$	55	24,41	218,219,220, 221	66,82
$15^{6}-15^{11}$	51	29	216,217	72
$16^{0}-16^{5}$	47,50,52, 56,57,58,	34,37	214,215	79
$16^{6}-16^{11}$		20,26,31,35	212,213	63,70,76,80
$17^{0}-17^{5}$	48,53	28,33,39		65,67,71,81
$17^{6}-17^{11}$	46,54,59		210,211	64,74,75,77
$18^{0}-18^{5}$		30	208,209	68,73,78,83
$18^{6}-19^{0}$			207	62
Total	16	15	19	21
Mean	16.1	16.3	16.1	17.2
Std. Dev.	1.5	1.0	1.5	1.0

Rather than refer to each pupil by name a number was assigned. It is that number that is used here.

The schools of Category B, C and D with mean mental ages of 16.1, 16.3 and 16.1 are close enough together that for practical purposes they might be considered equivalent. School A, however, has an average Binet Mental Age approximately one year higher than the other three.[1] That this higher mental age is the cause of the superiority of School A does not necessarily follow. School B is superior to Schools C and D even though it has no higher mental age. No conclusions concerning the relation between mental age and attainment on these tests will be made at the present. Instead Schools B and D which have identical mean mental ages will be compared to attempt to determine the cause of School B's superiority.

1. In the balance of the Chapter, by A B C and D Schools will be meant the Schools chosen as typical for each of these categories. For example: when the B School is referred to, it will be the results of School IV that are used.

Comparison of School B(IV) and School D(X).

Chart III reminds us that School B is, in general, superior to School D. Only in section (a) of Test IV Part I is School D superior ; and even in this test, if sections (a) and (b) are considered as giving but one score, then School B is superior. What is the reason for this general superiority? Already we have seen that these schools are equivalent in regard to mental age. The two schools may now be compared in regard to certain factors that might explain the superiority of School B over School D.

Sex.- Commonly we hear the claim that boys "take to" mathematics while girls do not, or, more cautiously, that boys are better at mathematics than are girls. An examination shows that Category D School has 9 boys and 7 girls while School B has 8 boys and 11 girls. School D thus has the higher percentage of boys so apparently sex offers no explanation of the difference.

Year in school.- At first the year in high school seemed to offer a logical explanation of the superiority especially when it was found that 8 of the School B students were in their second year of high school while all the School D pupils were in their first year of high school. This would mean that educational maturity or experience was a contributing factor in a pupil's success with a first course in Geometry. Added to the advantage of a year's experience in high school these eight students had all taken Algebra I in their first year of

high school; none of the School D pupils were taking, or had taken, Algebra I. Examination of the course in Algebra I would seem to make it clear that it would help in the Geometry course. However, further checking on the achievement of the eight second year pupils on the individual tests showed their work was not above the average of School B so could not account for the general superiority of the entire school. For example in Test I the average School B score is 13.4 questions. The eight students in their second year averaged only 12.3. On test II these same eight students averaged 57.7 to a School average of 60.1 questions. Years in school, then, does not explain the superiority of School B.

Other items on the record sheet.- A further examination of pupil record sheets shows the two schools to be practically equivalent in several respects which might affect ability in Geometry or in logical reasoning in general. In the first place, levels of past achievement in school (measured in terms of the distribution of A and B ratings on Grade IX examinations) are not fundamentally different. The total number of science courses, courses supposed to develop scientific attitude, which the students have taken, or are taking, show little difference in the two schools. Nor is there a noticeable difference in the number of students who have taken or are taking courses such as English or Social Studies.

The teacher questionnaire.- It was felt that the point of view, the methods and the aims of the teacher might be a determining factor in pupil success on these particular tests. If the teacher was cognizant of the features of Geometry that these tests stress, then his pupils should do better than the pupils of the teacher that merely "covers the text". Details in the matter of teacher methods, aims, etc. were called for in the questionnaire which the teacher was asked to complete. A copy of the questionnaire follows.

An examination of the completed questionnaires sent in by the teachers of Schools B and D reveals no greater differences than did the comparison of the record sheets of the pupils of the respective schools. Both teachers have a background of University training on which to base their teaching. Both teachers believe, firstly that Geometry instruction is useful if it develops a pattern of thinking that will apply in other subject-matter fields, and secondly that one on the means of attaining this wider usefulness is to tackle new problems which the pupils raise. Neither teacher considers the study of Geometry to constitute a memorization of formal proofs and constructions. There is no method available to compare the actual classroom procedures and techniques of these two teachers but as far as their answers to the questionnaire reveal their training, attitude and methods, no difference is apparent that might explain the superiority of School B.

QUESTIONNAIRE

The study in geometry in which you are participating aims to discover, elected provinces, what geometric concepts have been mastered by upils, and how well the pupils can reason in geometric and non- tric material. To throw light on these questions it is necessary ow what references teachers are using, the methods teachers use esenting geometry in the classroom, and the like. With this in you are requested to fill in the following questionnaire.

oes each student have a copy of a text? ______.

Name of text student uses. Author.

______________________________ ____________

______________________________ ____________

oes the student have access to any reference books in geometry? _____
f so, do students use such references: ____ frequently;
___ sometimes; ____ seldom; ____ never? (Please check).

Titles and authors of reference books in your classroom:

____

____

____

____

____

____

o you regularly use any reference books? ___________.

f so: Titles:	Use : frequently?	sometimes?
______________________	________	________
______________________	________	________
______________________	________	________
______________________	________	________

ave you read any of these reports: Please check, and give date of ast reading.

Date	Reports read (check)	
____________	_____	a. "The Teaching of Geometry in Schools". (Bell and Sons, London, 1930)
____________	_____	b. "A Second Report on the Teaching of Geometry in Schools"(Ditto, 1938).
____________	_____	c. "The Teaching of Geometry". (N.C.T.M. 1930 yearbook)
____________	_____	d. "The Nature of Proof" (Ditto, 1938).

o you read any of these journals?

Journal:	Read: Regularly	Sometimes
The Mathematics Teacher"(Journal of N.C.T.M.)	______	______
Mathematical Gazette" (G. Bell and Sons.)	______	______
School Science and Mathematics".	______	______
Others:		
______________________	______	______
______________________	______	______
______________________	______	______
______________________	______	______
______________________	______	______

hat are your special qualifications for the teaching of mathematics? University training, special summer courses, etc.)

____

____

____

____

____

n addition to geometry, what other subjects do you teach?

____

____

ou are asked to examine the following quotations to see which most losely describes your own ideas of the aim of teaching geometry. hen indicate your first choice with a 1. If you consider the others ould be second and third best, indicate your preference. Please give our opinion in this way. A space is provided below for any comments bout your own aims you may wish to make.

A. "Geometry achieves its highest possibilities if it can establish a pattern of reasoning; if it can develop the power to think clearly in non-geometric situations; if it can develop an appreciation of the place and function of definitions and assumptions in the proof and conclusion. Such are the aims of geometry teaching today."

B. "In human life today, as in the past, geometry has played a great part. The history of this contribution to civilization has itself great cultural value, but coupled with the mastery of useful geometric concepts so necessary for advanced work in any of the sciences, it provides the complete justification for the teaching of geometry in the schools."

C. "The chief value of geometry lies in its power to train the mind. The rigid kind of teaching involved in proving propositions in geometry trains the mind to be critical and rigorous in thinking. It is the development of this habit of logical thinking that is the chief aim in the modern teaching of geometry."

My choice: A.____ B.____ C.____

omments: ____

____

____

____

hat outcomes in pupil behaviour do you expect as a result of the teaching of geometry? Opposite the outcome or outcomes you consider most important place a 1, opposite the outcome or outcomes next most important a 2 and so on. If you consider some of these outcomes not worthwhile leave the space blank.

a) The student will be able to prove a certain number of propositions.

b) He will recognize the significance of defined and undefined terms for any argument.

c) He will be able to solve geometric problems (deductions).

d) He will be able to solve more or less original geometric problems (i.e. where the geometric applications are not stated in the problem situation).

e) He will require evidence for any important conclusions he is asked to accept.

f) He will have acquired the habit of thinking by carefully graded steps as in the deductive proofs of geometry.

g) He will have obtained some idea of algebra applied to geometry and vice-versa.

h) He will have some idea of space geometry.

i) He will have mastered a number of geometric concepts which are the necessary prerequisite for advanced work in mathematics.

j) He will appreciate the contributions of mathematics to the progress of civilization.

o you feel you are realizing your aims and outcomes? Very little____; omewhat ____; fairly well ____. (Please check)
hat are the obstacles?

____

____

Are your students required to keep notebooks?______. If so, do you check these periodically? _______. Do your students use workbooks?_____ If so, what is the name and author of the workbook?

Do you require students to do and hand in assignments (e.g. problems to solve)? Regularly and often ______; sometimes ______; never ______ (please check).

Here are a number of statements often keenly disputed by teachers of geometry. Considering your own classroom methods as a basis for judgment, mark these: A--You agree with the statement.
U--You are undecided.
D--You are inclined to disagree with the statement.

a) The best way to make geometry interesting and useful is to provide the students with many real life problems which apply geometric principles.
b) Students should be required to stick to one method of solving a problem. Individual proofs, all different, even though logical, will confuse the students and defeat the aims of geometry teaching.
c) When a new principle occurs in the study of geometry, it should be discussed in terms of the students' life situations, and for other studies if possible, e.g. the baseball game, the street corner argument, physics problems. Such a procedure is not really a waste of time.
d) In a subject like geometry, class discussion is largely a waste of time. Students learn much more by watching demonstrations or doing them themselves.
e) A geometry textbook should always contain answers to numerical problems so that students can try again if they get the wrong answer.
f) Students often want to tackle new problems which would sidetrack the main trend of your day-by-day work. A teacher must prevent this.

Please check your view:

g) The proofs of none ____; all ____; some ____; of the text theorems should be memorized.
h) At times ____; at any time ____; at no time ____; should students be allowed to work in pairs or groups when solving problems.

If pupils have taken or are taking courses, other than courses in mathematics and the sciences, in which some attention is paid to the principles of logical reasoning, please name these courses, with any necessary comment.

Study of Progress of Pair of Schools.

School B (IV) and School D (X)

Table X - Results of Schools B and D on Test I.-

The number correct out of a total of sixteen questions in Test I is recorded for each student of Schools B and D. The students are arranged in order of decreasing mental age. (See Table IX) .The mean score of the 18 students of School B on Test I is 13.4 with a standard deviation of 2.24, while the mean score of the 14 students of School D is 10.4 with a standard deviation of 2.74. Thus on the average a B student got 3 more questions correct than a D student.

Also worthy of note is the fact that 16 of the B students (89%) scored above the D school mean score while only 7 of the D students (50%) scored above this mean score. Also 10 B students (56%) obtained scores above the B School mean score while only 3 D students (21%) excelled this mean score.

The results indicate greater instability on the part of School D. Not only do the scores range from 4 to 14 in contrast to a range from 8 to 16 for School B but also School D has a larger standard deviation.

Before proceeding with a search for the cause of the difference between the scores of School B and D on Test I it is necessary to ask the question, "Is this difference significant?" That is what chance would there be to get as large a difference in random samples of students taken from the same parent population.

TABLE X

SCORES OF SCHOOLS B AND D ON TEST I (POSSIBLE 16)

School B - 18 students		School D - 14 students	
Pupil	Score	Pupil	Score
207	16	46	10
208	15	54	11
209	10	59	14
210	15	48	12
211	15	53	7
212	12	57	14
213	15	47	8
214	11	50	11
215	16	52	10
216	13	51	14
217	15	55	4
218	12	49	9
219	8	60	10
220	13	61	12
221	14		
222	16		
223	11		
224	14		
Total	241	Total	146
Mean	13.4	Mean	10.4
Std. Dev.	2.24	Std. Dev.	2.74

To determine if the difference between the variances is significant we use the technique outlined by Kenney.[1] The difference in variance between the two samples gives a value of F equal to 1.52. Since the tables indicate 5 chances in 100 of getting a value of F as large as 2.35 the chances of a value as large as 1.52 are even better. Hence we can say the difference of variances is not significant; the observed instability of scores for School D might readily be entirely chance results.

The Fisher t Test is used to determine the significance of the difference of means.[2] The value obtained for P is less than .004. This shows that there are less than four chances in a thousand that two random samples from the same parent population would have a difference as large as 3.0. Thus the difference appears significant. What factors might account for the difference?

---

1. John F. Kenney. Mathematics of Statistics - Van Nostrand Co. Inc.
Part II Page 145 and Table II
Estimated variance of Parent Population using School B sample is
$\frac{18}{18 - 1} \times 5.05 = 5.35$

Estimated variance of Parent Population using School D sample is
$\frac{14}{14 - 1} \times 7.56 = 8.14$

Then $F = \frac{8.14}{5.35} = 1.52$, but P(.05) and P(.01) give in Table II 2.35 and 3.40 as values of F.

2. Yule and Kendall - An Introd. to Theory of Statistics
Page 442 and Table V.
$t = 3.29$ $\quad v = 30$ $\quad$ P is less than .004.

Table XI - Results of Schools B and D on Test II.-

The comparison of Schools B and D has yet revealed no factor or factors that might account for the general superiority of School B or for School B's significant superiority of 3.0 questions out of sixteen in Test I. It has been shown that the two schools are equivalent with regard to mental age, sex, subjects taken, and nature of the instruction received. The possibility of a relation existing between a student's grasp of the logic of the problem and his ability to handle the problem being the basic enquiry of this study, the schools were examined from this point of view.

Being entirely objective in nature, Test I can yield little information concerning the details of the reasoning process used by the pupils. Some of the other tests will needs be examined to investigate the possibility that the difference between the achievement of the pupils in the "typical" school of category B and the achievement of the pupils of category D's type school is due to a difference in the form of reasoning used to attack the given problem.

The results of Test II are first studied. In Table XI the number of blanks correctly filled , out of a possible of 79, is entered for each pupil of the two schools. These scores are totalled; then the means and standard deviations found. The significance of any differences in means or standard deviation between the two samples is checked by the same techniques outlined in the handling of Test I results.

TABLE XI

SCORES OF SCHOOL B AND D ON TEST II (POSSIBLE 79)

School B - 17 students		School D - 16 students	
Pupil	Score	Pupil	Score
207	69	46	60
209	59	54	68
210	66	59	71
211	69	48	51
212	57	53	54
213	68	56	32
215	64	57	50
216	50	58	47
217	60	47	72
218	62	50	47
219	58	52	44
220	63	51	65
221	58	55	50
222	69	49	53
223	55	60	47
225	50	61	62
224	45		
Total	1022	Total	873
Mean	60.1	Mean	54.6
Std. Dev.	7.03	Std. Dev.	10.7

As the reader might expect if he glances at Chart II Page 55, the superiority of School B over School D is not as great in Test II as it was in Test I. School B pupils average 60.1 questions correct out of 79 to School D's 54.6 questions correct. However, this average superiority of 5.5 questions on the part of School B pupils is much less significant on 79 questions than was the superiority 3.0 questions per pupil for the 16 questions of Test I.

It might be noted that 8 B students (47%) excelled the B mean score while only 5 D students (31%) excelled the score. Similarly 14 B students (82%) scored above the D mean score, while only 6 D students (37%) exceeded this score.

Also interesting is the greater instability of the D students where the standard deviation is 10.7 compared with 7.03 for B students. This same effect may be observed by noting the wide fluctuations of scores for D students, particularly at the lower part of the table where the students of lower mental age are listed.

In spite of these apparent differences between the performance of the two schools on this test the checks for significance of difference of both mean and variances are applied to determine the probability of obtaining as large differences in random samples. In the case of variances

[illegible]

[illegible]

[illegible]

[illegible]

even if these were samples from the same parent population there would be slightly better than 5 chances in 100 to get standard deviations as widely separated.[1] Fisher's t Test shows that under similar conditions, the probability of obtaining by random sampling one sample with mean as much as 5.5 above a second sample's mean is .048.[2] Thus we must conclude that School B's superiority does not show up as highly significant for this test.

Still certain conclusions can be made from the fact that School B's superiority is less on Test II than it was on Test I. Since Test II outlined the solution of the problem, it is apparent that the Category D pupils on the whole, do relatively better if they are given certain hints regarding the steps in the proof. If the outline solutions given to the problems on Test II are the reason for D students' better showing on Test II, then it follows that their poorer showing on Test I is a result of their inability to develop themselves a logical solution either because of a lack of geometric knowledge or because of a failure to see the applicability to the problem of the knowledge possessed. An attempt will be made in the subsequent work to determine which of these causes operate.

---

1. $F = 2.31$ Table II gives 2.39 and 3.48 as the .05 and .01 points of significance.

2. $t = 1.71$ $v = 31$
Probability of B School exceeding D school observed amount in chance sampling is .048.

<u>Tables XII and XIV - Results on Corresponding Problems of Tests I and II.-</u> As w s stated on page 12, "the exercise of Test I[I] can almost completely be paired with exercises of Test I requiring the same concept." If a student is successful in a Test I exercise and not on the Test II exercise involving the same concept, then the only explanation that seems possible is that the student has not developed to the stage where he can put his thinking into formal pattern. If a student is successful on a Test II item and not on the Test I item then he is apparently able to fill in an outlined proof but not able to develop it for himself. The distribution of errors in Test I is shown in Table XII, for both B and D students.

TABLE XII

ERRORS ON QUESTIONS OF TEST I

Blank	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
% of 14 D st making errors	64	0	21	29	14	7	50	14	21	50	29	21	0	29	14	36
% of 18 B st making errors	17	0	0	33	22	0	17	11	28	22	6	11	0	17	11	28

Since B School did better on Test I than D School it should not be surprising that a higher per cent of the D students made errors on most of the questions. In Table XIII an attempt is made to pair the questions of Test I - particularly those questions where D students seemed to have trouble--with questions of Test II. The principles involved in the paired questions are also included in this table.

TABLE XIII

MATCHING QUESTIONS OF TEST I WITH QUESTION OF TESTII INVOLVING SAME PRINCIPLE

Test I		Test II		Principles or Principles Involved
Exer.	Blank	Exer.	Blank	
I	(1)	IV	39,40,41	If two triangles are similar then sides are proportional.
III	(3)	II	12	Hypotenuse of right triangles by Pythagorean theorem.
IV	(6)	II	18	Find base angle of right isosceles triangle.
V	(7)	I	(6)	The two acute angles in right triangles are complementary. The two angles making a straight angle are supplementary.
VI	(8)	II	(18)	Base angle of isosceles triangle given; find vertex angle.
VII	(9)	III	(22)	Alternate angles by parallel lines are equal.
VIII	(10)	I	(2)	In a right angled triangle the two acute angles are complementary.
IX	(14) & (15)	VI	(66)(67) (68) #	Relation between chord and perpendicular diameter in circle.

Test II question, while requiring the same basic facts, is simpler than the Test I blanks.

Before examining the performance of individual students with regard to each of the matched problem pairs of Table XIII the comparative success of Schools B and D on the matched questions is tabulated. The figures regarding Test I are a duplicate of Table XII reproduced here to facilitate comparison.

TABLE XIV

DISTRIBUTION OF ERRORS BETWEEN B AND D STUDENTS ON MATCHED QUESTIONS FROM TEST I AND TEST II.

Test II	Blank	39	40	41	12	18	6	18	22	2	66	67	68
	% D Errors	81	31	44	25	13	50	13	25	13	6	6	6
	% B Errors	41	88	12	18	0	6	0	12	0	6	0	12
Test I	Blank		1		3	6	7	8	9	10		14	15
	% D Errors		64		21	7	50	14	21	50		29	14
	% B Errors		17		0	0	17	11	28	22		17	11

From the top three rows of figures it is evident that the D students made more mistakes than did B students on these questions of Test II.[1] The uniformity of the superiority even more than its numerical value is the feature which tends to indicate a reliable difference; for if no difference in ability existed between B and D Schools then each school would tend to excell in approximately one-half of the questions.

1. Two exceptions, questions 40 and 68. Former needs investigation.

The nature of Test II should be recalled. Requiring the student, in the simpler cases, to fill in the specific implications of an indicated theorem, this test supplied to the student the advantage of knowing which theorem to apply to the problem. Or, in other cases, it was required that the reason (or theorem) supporting a given statement be filled in the blank. As was indicated in the description of Test II correct responses required not only comprehension of the theorem and its implications but also an ability to record correctly this comprehension. In reality the obtaining of an answer in Test I would appear more difficult as the student had to find and decide on the theorem to be used. However Test I was easier inasmuch as the student did not have to give a logical statement of his reasoning -- just get the answer.

With regard to a pair of questions --- one in Test I and one in Test II --- based on the same fundamental principle there are the following possibilities:-

1. Test I question correct; Test II question correct.
   -indicating both a comprehension of the principle and an ability to use it constructively in a problem.
2. Test I question correct; Test II question wrong.
   -This indicates an ability to use the theorem but an inability to express this principle logically.
3. Test I question wrong; Test II question wrong.
   -meaning a failure to understand the principle and hence naturally an inability to use it or state it.

4. Test I question wrong; Test II question right.

This would mean an understanding of the principle but lack of power to see its application.to a problem.

Since we are trying to find the reason for the statistically significant inferiority of School D on Test I we are interested primarily in those cases where the Test I question is wrong - classes (3) and (4) above. The errors made on the matched questions of Test II were not necessarily made by the same students making errors on the Test I question. Table XV shows how the students who made errors on Test I questions did on the Test II questions.

For example, 9 D students erred on Blank 1 Test 1. On the corresponding questions of Test II 6 of these students gave wrong answers and 3 gave correct replies.

Similarly the other pairs were examined, the results being tabulated in Table XV.

It must be kept in mind that while the number of students in the sample is small, the sample is in no way unusual. In fact it is hoped that each group is a fair sample of the category to which it belongs. Nor are the questions unusual, except that they can be paired in Test I and II.

TABLE XV

DISTRIBUTION OF RIGHT AND WRONG ANSWERS ON
TEST II QUESTIONS CORRESPONDING TO TEST I QUESTIONS
(RESULTS ONLY FOR STUDENTS ERRING ON TEST I)

Pair of Questions		D students making errors on Test I member get Test II member	
Test I	Test II	Right	Wrong
1	39,40,41,	3	6
3	12	1	2
6	18	1	0
7 .	6	1	6
8	18	2	0
9	22	3	0
10	2	7	0
14,15	66,67,68	5	1
Total		23	15

Of 38 errors made in Test I by D students, in 23 cases the student indicated, by a correct answer on the corresponding Test II question, that he understood the principle involved. In 15 cases he did not understand the principle. The figures of Table XVI lead to certain tentative conclusions:-

1. Students' not being able to correctly apply a principle or theorem to a problem is responsible for more errors than is students' not understanding the theorem or principle.

2. Certain schools obtain poorer results than other schools, not because the pupils do not understand the geometry theorems involved but because the students have not progressed sufficiently to be able to see when a theorem is applicable and how applicable to a specific problem. This would seem to imply that the students can give correct responses in questions on geometrical reasoning but they can not themselves proceed using the reasoning process. This might mean that the student has progressed to the stage where he understands a geometrical concept but can not go ahead himself to use it; or it might indicate that the student has memorized responses which he does not understand and which are meaningless to him.

These tentative conclusions will be checked as the study progresses. In the first place the achievement of the Schools B and D on Tests III and IV will check the conclusions. Also the examination of the paired samples from the typical schools of Categories A and C will serve as a parallel analysis.

Results of Schools B and D on Test III.- Test III called for the student to give the entire solution himself. With a view to getting greater uniformity in the solutions - a uniformity designed to facilitate marking - the page was ruled with special sections for the work. A place was supplied for the figure along with instructions regarding the labelling of the figure. To assist the student to think in terms of the geometrical facts of the problems, which were not stated in geometrical terms, space was provided where the student was

asked to list the items of information supplied in the problem. The solution itself was to be placed, a step to a line, with the reason for the statement to be placed immediately following the statement. The same set of reasons that was used in Test II was used in this test, consequently the student was already familiar with the list thus enabling him to work more rapidly.

The marking of Test III was more laborious than the marking of the two previous tests. There were three respects in which the test was marked. First the number of answers correct was found. In five exercises there were a total of seven answers required. The number of these answered and the number of correct answers were both recorded. Then as a second score the student was marked on his transfer of the information of the problem into geometrical terms as indicated by his figure and his listing of the data under the heading "given". It was not required that the student give the information both in the figure and in words but merely that he indicate he had realized what geometrical information was available with which to work. With a mark of five assigned to each question the total possible score for "Interpretation" was 25. The last score on this test was given for the solution itself. Here again five marks were assigned to each exercise, a perfect score being given only if the student had correct statements and named the right theorems as authority for each statement. The scores in the individual questions and the totals are given in Table XVII for the students from Schools B and D.

TABLE XVI

SCHOOL D RESULTS ON TEST III (16 STUDENTS)

Pupil	Answers - Possible 7									Interpretation Possible 25						Solution Possible 25					
	I		II	III	IV	V		At	C	I	II	III	IV	V	T	I	II	III	IV	V	T
	1	2				1	2														
46	.	.	.	.	.	.	.	0	0	5	5	4	2	4	20	2	0	0	0	0	2
54	1	1	.	.	.	.	.	2	2	4	0	2	1	0	7	4	0	0	0	0	4
59	.	1	1	.	1	.	.	3	3	5	5	0	5	5	20	2	5	0	5	1	13
48	1	x	.	.	x	1	x	5	2	5	1	0	5	5	16	1	0	0	2	2	5
53	x	1	.	.	1	.	.	3	2	5	3	2	4	5	19	0	0	0	4	0	4
56	.	.	x	.	.	.	.	1	0	5	0	.	0	0	5	1	0	0	0	0	1
57	.	.	.	.	.	.	.	0	0	5	4	2	0	5	16	2	0	0	0	0	2
58	.	.	x	.	.	1	1	3	2	5	5	3	0	5	18	3	3	1	0	3	10
47	x	1	1	x	.	.	.	4	2	5	2	2	3	5	19	2	5	1	0	0	8
50	x	x	1	x	.	x	1	6	2	5	5	1	0	5	16	0	0	0	0	0	0
52	.	.	.	.	.	.	.	0	0	5	5	2	0	4	16	3	0	0	0	0	3
51	1	1	x	.	1	.	.	4	3	5	5	5	5	5	25	3	4	0	5	0	12
55	.	.	.	.	.	.	.	0	0	4	1	.	4	.	9	2	0	0	1	0	3
49	.	.	.	.	.	.	.	0	0	5	.	2	0	5	12	2	0	0	0	0	2
60	x	x	.	.	.	x	x	4	0	5	.	θ	0	1	6	1	0	0	0	0	1
61	1	1	.	.	.	x	1	4	3	4	0	0	0	5	8	3	0	0	0	2	5
Total								39	21						233						75
Mean								2.4	1.3						14.6						4.7
Std. Dev.								2.0	1.2						5.7						3.9

1 -- question correct

. -- question unanswered

x -- answer wrong

TABLE XVII

SCHOOL B RESULTS ON TEST III (19 STUDENTS)

Pupil	Answers - Possible 7									Interpretation(25)						Solution(25)					
	I 1	I 2	II	III	IV 1	IV 2	V	At	C	I	II	III	IV	V	T	I	II	III	IV	V	T
207	1	1	1	1	1	1	.	6	6	5	5	5	5	5	25	4	5	5	5	3	22
208	1	1	1	.	1	.	.	4	4	5	5	4	5	5	24	1	5	1	4	0	11
209	.	.	1	x	x	.	.	3	1	5	5	5	5	5	25	1	4	4	4	0	13
210	1	1	1	x	1	x	x	7	4	5	5	5	4	0	19	4	5	4	3	0	16
211	1	1	1	1	1	1	1	7	7	5	5	5	5	5	25	2	5	5	5	4	21
212	1	1	1	1	1	.	.	5	5	5	5	5	5	1	21	5	5	5	5	0	20
213	1	1	1	.	.	.	.	3	3	5	5	1	4	0	15	4	4	0	0	0	8
214	1	1	1	x	1	x	x	7	4	5	4	0	4	5	18	3	4	0	3	0	10
215	1	1	x	x	1	.	.	5	4	5	5	0	5	5	20	4	4	0	4	0	12
216	1	1	.	1	.	1	1	5	5	5	4	5	0	5	19	2	1	0	0	3	6
217	1	1	1	x	x	1	x	7	4	5	5	5	4	5	24	3	4	1	4	2	14
218	1	1	x	x	x	.	.	5	2	4	4	2	5	2	17	0	2	2	1	0	5
219	1	1	x	1	1	.	.	5	4	5	5	5	5	5	25	3	1	3	5	0	12
220	1	1	x	1	1	1	1	7	6	5	5	5	5	5	25	5	4	5	5	3	22
221	1	x	x	x	1	.	.	5	2	5	0	2	5	0	12	3	0	0	4	0	7
222	1	1	1	1	1	1	x	7	6	5	5	5	5	4	24	5	4	5	3	0	17
223	1	1	x	.	x	.	.	4	3	5	0	0	4	5	14	2	0	0	1	1	4
224	1	1	x	.	1	.	.	4	2	5	3	1	4	3	16	4	0	0	5	0	9
225	1	1	x	x	.	.	.	4	2	5	5	2	4	.	16	2	3	0	1	.	6
Total								100	74						384						235
Mean								5.3	3.9						20.2						12.4
Std. Dev.								1.4	1.6						4.3						5.8

1 - question correct
. - question unanswered
x - answer wrong

The results on Test III as revealed in Tables XVI and XVII are very illuminating. In School D the pupils secured answers in only 39 cases, or an average of 2.4 questions per pupil. Five D students had no answers at all. In contrast each B student had three or more answers; also on the average each B student had answers to 5.9 questions. Of the 39 questions completed by D students 21 or 54% were correct giving an average correct per pupil of only 1.3. Seventy-four per cent of School B's one hundred completed questions were right giving an average of 3.9 questions per pupil. An application of the t test shows the superiority of School B to be highly significant both in regard to answers obtained and answers correct.[1] The number of answers obtained whether correct or wrong was included since it seems to be a mark of superiority if the pupil can get as far as an answer. That is more credit goes to the student who can follow a question as far as an answer even though, through possibly a small mechanical mistake, he does not end up with the right answer. Certainly the number of correct answers is a measure of ability in this test. Then summarizing we have the scores as follows:-

ANSWERS

School	Number Questions Completed				Number Questions Correct			
	Total	Mean	Std.Dev.	Range	Total	Mean	Std.Dev.	Range
B	100	5.3	1.4	3-7	74	3.9	1.6	1-7
D	39	2.4	2.0	0-6	21	1.3	1.2	0-3

1. Probability of obtaining differences by chance is less than .0005 for each.

The superiority of School B in this part is shown by
(1) significantly higher mean scores per pupil,
(2) greater stability of scores (differences of variances not statistically significant) and
(3) the higher ranges in which scores fall
seems equally as outstanding as that found in Test I. Can the reason for the superiority be found or analyzed in the other totals of the test?

The performance of the two schools on Interpretation can be summarized as follows:-

INTERPRETATION

	Total	Mean	Std. Dev.	Range
School B	384	20.2	4.3	12-25
School D	233	14.6	5.7	5-25

Tests of significance showed the mean for School B as significantly superior to the mean of School D.[1] The difference in standard deviations is not significant. However in the matter of mean score per pupil the difference on interpretation is smaller than that obtained in the case of answers. This might be interpreted to mean that the inferiority of D students is not due to an inability to understand the terms involved in the problem. This would be as was hoped, for the problems were devised with the intention that all students undertaking them should be familiar with the vocabulary used and that any difficulties would then lie not in the understanding of the

1. Probability less than 0.004

problem but in the geometry of the problem. The results seem to show that this object has been achieved with a fair amount of success, for in the matter of answers the B School outscored the D School almost four to one, while in the matter of interpretation School B scored only approximately half again as high as D School. Certainly then this difference in ability to interpret the problem will not account for the total difference in achievement of the two schools.

The results on "solution" show greater differences. The marks of School B ranging from 4 to 22 average 12.4 per pupil with a standard deviation of 5.8; while the marks of School D ranging from 0 to 13 average 4.7 per pupil with a standard deviation of 3.9. This difference by the t - test shows School B significantly superior.[1] What else can be implied from these figures? It has already been suggested that the small difference in regard to interpretation indicates that the difference between the two groups in geometric achievement can not be accounted for by saying that the B group has seen more of the world about them, is more familiar with the objective world in which they live and as a result can see meaning in the problems which are "over the head" of the less "worldly-wise" D students. This is often a reason given for the lack of success that some students have in the study of geometry. The score seems to discard this explanation.

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1. Probability less than .0005

If we draw the above negative conclusions from the small difference between the scores of the schools in the matter of interpretation to be consistent, then, we must conclude that the larger difference in the matter of Argument or Solution offers a plausible explanation of the difference in geometric ability of these two groups of students, which we have selected as typical of much larger classes. It might be carrying the results too far to say that the inability to reason is the CAUSE of the lower scores. Safer it would be to say that the inferiority of the second group shows up at this point -- a point one step closer to the cause than the mere realization that the one group is superior to the other. A glance at the number of zeros in the solution part of Table XVI shows that the D students were completely baffled as to the method of attack to use on many of the problems. Actually there were 51 problems in which the D students did not even get started on a solution. This in contrast to 25 problems for the larger number of B students.

The conclusion here suggested corroborates the conclusion that was drawn from the results on the schools on matched problems of Tests I and II. The conclusion was that the student fails to solve the problems because they are not "able to see when a theorem is applicable and how applicable to a specific problem," not because they do not understand the problem or the theorem.

The examination of the results of some students seems to cast certain doubts on this conclusion which we have just stated. For example, Student 50 obtained the correct answer to Exercise II, he obtained full marks on interpretation but he received no marks for his solution. Student 216 does the same thing on Exercise III. While these cases are the exception rather than the rule, they apparently contradict the conclusion that was stated above. Various explanations might be offered. The sceptic might say that these students succeeded in obtaining the answer in spite of the cumbersome technique which we impose. Less rash would be the explanation that the students were using the methods of logical reasoning subconsciously but still were not able to put their thinking processes on paper. Another possible explanation is that the student had a good neighbor who helped him to the extent of giving an answer. Only the one explanation contradicts our conclusion, in contrast to the many explanations which can explain the phenomena in accordance with the hypothesis. Consequently till more information is found the tentative conclusion will be allowed to stand.

In summary then, Test III seems to support the hypothesis that deficiencies in problem solving ability in geometry are usually due to the student's not being able to proceed with a logically developed solution from the data that is supplied.

In the majority of cases this difficulty stops the student even though he does understand the problem that confronts him. At this stage there is no pedagogical remedy suggested to overcome the obstacle or any inference made as to the difference in training which has caused the deficiency.

Results of School B and D on Test IV.- Two tentative conclusions have been derived. The first, based on Test II results, states that there is very little difference between groups of students of comparable ages in regard to their understanding of the theoretical principles of logic. Thus, if two student groups are asked specific questions on the theory of logic they score approximately the same. The second conclusion that has been stated, subject to revision in the light of future findings, has to do with the ability of the student to apply known theoretical concepts to the solution of the actual problems which he encounters in the world around him. It appears that this is the aspect of problem solving that offers the greatest difficulty, and hence the one that most frequently accompanies inability to solve problems. This hypothesis was first stated as a result of Test II and verified in the results of Test III.

Test IV is divided into two parts. The one part deals entirely in the abstract, theoretical concepts of logic; and the other deals with the application of these same concepts to actual problems, some of the working man variety, some of the

[illegible]

[illegible]

class room variety. Thus Test IV should offer a further basis to check the above conclusions. If the conclusions are to be borne out in this test, then the difference between Schools B and D should be small on Part I of the test where the students need only recognize certain principles of reasoning. On Part II where the students must apply these principles we should expect a larger difference.

The results on Test IV are tabulated in Table XVIII. The terms "Correct", "Guess", "Wrong", "Conclusions Correct", "Reasons Correct" and "Irrelevant Reasons" are used in the senses that have already been described.[1]

1. See Page 46 and Page 49.

[illegible]

[illegible]

[illegible]

TABLE XVIII

ACHIEVEMENT OF SCHOOLS B AND D ON TEST IV.

D students							B students						
Pupil	Part I			Part II			Pupil	Part I			Part II		
	C	G	W	CC	RC	IR		C	G	W	CC	RC	IR
	Possible 17							Possible 17					
46	14	1	2	5	7	4	207	15	1	0	9	10	2
54	14	1	2	7	8	5	208	13	2	1	8	6	0
59	13	2	2	7	6	4	209	15	2	0	8	9	1
48	5	9	1	8	10	20	210	11	1	2	8	13	6
53	11	5	1	6	7	5	211	9	5	3	9	9	1
56	5	9	3	Missing			212	12	4	1	7	9	10
57	11	6	0	7	8	7	213	10	5	2	9	7	4
58	13	3	1	7	9	7	214	8	4	2	6	2	2
47	9	5	3	5	8	4	215	13	2	2	6	10	4
50	11	3	3	7	9	14	216	7	7	1	7	3	4
52	9	7	1	4	10	10	217	10	4	2	6	6	4
51	13	3	1	5	6	3	218	4	2	2	7	7	2
55	12	3	2	4	6	9	219	12	3	1	7	8	9
49	10	6	1	6	6	5	220	9	5	3	8	8	6
60	8	8	1	6	5	8	221	7	4	0	10	6	4
61	7	6	3	5	6	10	222	8	5	4	8	12	5
							223	6	4	2	8	11	2
							224	8	6	3	5	7	3
							225	8	2	4	8	9	3
Total	165	77	27	89	111	115		185	68	35	144	152	72
Mean	10.3	4.8	1.7	5.9	7.4	7.7		9.7	3.6	1.8	7.6	8.0	3.8
Std. Dev.	2.9	2.5	0.9	1.2	1.6	4.4		3.0	1.7	1.2	1.3	2.7	2.5

C - Correct
G - Guess
W - Wrong

C.C. - Conclusion Correct
R.C. - Reasons Correct
I.R. - Irrelevant Reasons

Testing for significance shows that neither the differences in means nor the differences in standard deviations are of significant magnitude in Part I of Test IV. There is no superiority on the part of either school. That is the two schools are equal with regard to ability to respond correctly to general questions on the theory of logic. Inasmuch as School B has been superior in the logic of geometry this may seem at first surprising. For teaching the results are evidence that the imparting of a general knowledge of theoretical principles is not enough to ensure an ability to apply these principles.

Part II shows a different picture. School B gets 7.6 conclusions correct per pupil to 5.9 correct per pupil of School D - a significant difference.[1] The slight numerical superiority of School B in correct reasons is not statistically significant. However, in including only 3.8 irrelevant reasons per pupil to 7.7 per pupil for School D, School B is definitely superior.[2] There are fewer than two chances in a thousand to secure such a difference by chance if the two samples were from the same parent population.

These figures imply that while School B was not superior on a test where the principles were left in general terms it was significantly superior when asked to apply these principles to actual problems. This would bear out the two conclusions stated earlier.

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1. Chance probability is less than .0005

2. Chance probability is less than .002

[illegible]

[illegible]

[illegible]

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(1) [illegible]

(2) [illegible]

Summary of Argument from the Study of Schools B and D.- Although all visible factors are equal for the two schools, still consistently throughout the tests one school is superior to the other. If it were a matter of chance, then one school would be superior on the one test; the other on the next test. The uniformity of School B's superiority and the magnitude of this superiority, especially in Test I, Test III (Answers) and Test IV Part II, makes it highly improbable that the difference is a matter of sheer chance or of random sampling.

Since a difference exists the next thing is to find just where this difference lies and if possible to find a cause of the difference, so that the study might be of some value to teachers of Geometry. Certain factors that are commonly mentioned when the aptness of a student at Geometry is being discussed have been eliminated. Some of these factors are I.Q., sex, years in school, likings as indicated by High School courses selected and lastly the type of instruction being received. The two samples are equivalent in all these regards as far as is determinable.

The results of the tests themselves showed that certain other factors were of little significance. The most surprising of these is the factor of score on a test dealing with actual book problems of geometry. To score highly on problems stated in geometrical terms and involving geometrical constructions and deductions does not ensure that a high score will be obtained where the knowledge must be applied to a problem

stated in non-geometric terminology. No more significant in determining pupil ability in this last type of problem is the score obtained on a set of questions dealing with principles of logic such as the If-Then argument, argument by exclusion, arguments involving converses and such fallacies of argument as personal attack, assuming a converse, arguing in a circle and the use of irrelevant reasons and false authority. A knowledge of these principles and fallacies sufficient to recognize the truth or falsity of statements dealing with them is not always accompanied by an ability to apply them to problems in such a way as to support an argument or to show the error of an erroneous argument. No more effect has a knowledge of theorems and constructions as Euclid first set them up, upon the pupil's ability to actually use these to solve the type of problems that might some day confront him. These conclusions are negative in nature.

The positive conclusions, that the better a student can apply his theoretical knowledge to problems the better he will do on these same problems seems almost obvious without a lengthy study to reveal it. But has it been so obvious in the past? Too often we have assumed that if the truth could only be driven into the pupil's hard head then automatically the student would be able to apply the knowledge to any situation. This study has discredited any such rationalization. It now becomes clear that the student must be given definite experience in the application of the knowledge to the type of situation in which he must use it. We must be very careful in stating that the principles that we are developing in geometry can be transferred to other subject matter fields. All that this

study can so far conclude is that actual knowledge does not mean useful knowledge.

It might be argued with some degree of truth that the student who can not apply a principle does not really KNOW it. This question has been evaded by using terms "know" and "knowledge" to indicate the state of mind when the student can respond correctly to the type of question we have put before him in Tests II and IV Part I. It might be a memorization of formal statements but it is merely a quibbling over the definition of a word to argue on this point.

One other comment should be made concerning the word usage in the study. Throughout the study, the tests, such as Tests I and III, calling for the application of concepts have been used as the scale according to which the student's ability has been measured. Why are not the other tests equally as reliable measures of this ability? Here the utilitarian concept enters into the discussion. When the student starts to make his way in the world he will be measured by the extent to which the things he knows helps him to adjust his life to his environment. Accordingly the idea that the most satisfactory measure of ability is the use to which it can be put has been used in this study.

A very encouraging observation can be made from the comparison of Schools B and D. In Chapter I the question was asked, "Can Geometry be given direct and practical usefulness, and at the same time give the pupil certain ideas about the nature of proof." The results of School B would give an affirmative answer to this question, for this school has excelled in the solving of problems and has at the same time indicated that it has mastered "certain ideas about the nature of proof" as thoroughly as has School D.

Study of the Results of School A and School C

School I was placed by itself in Category A, not only because it gave higher results than any other school but also because it seemed a bit unusual. When it is said that this school is unusual it is not implied that the students are super-normal; but rather it implies that the conditions under which they are being trained are perhaps better than average. A high percentage of the students have A ratings on their Grade IX exams, and all the students are in at least the second year of high school. This last fact will mean the majority have the advantages of having taken Geometry and geing chronologically one year older. Table IX also showed the average mental age to be one year above the other typical schools. By the comparison of Schools B and D we saw that difference between schools could exist without any of these factors being present. The advantages of School A however, must be kept in mind in judging the achievements of the two schools.

Besides these conditions in the student body the instruction is of a high quality at School A. There is no intention to belittle the training given in School C but rather to note that School A has a condition not existing in every school. The teacher in this school has not only a University background in Mathematics but in addition has a live interest in the relation between the reasoning processes in Geometry and

in the other subject fields. In response to questions on the teacher questionnaire he indicates that as principal of the school he is trying to get the other teachers of his school also to use their courses to develop in the students a realization of the significance of the principles of logical reasoning. This must be kept continually in mind in evaluating the results of this school. We will want to ask ourselves, "How well is he succeeding?" and "Is he sacrificing something else to achieve these results?"

When one considers these factors it is not surprising that School I is superior. The records of Schools A and C will not be examined in the same detail as were the scores of Schools B and D, but rather the results will be used to check the hypotheses that have already been stated regarding the nature of the superiority,one school over another.

In Table XIX the total score of each pupil of Schools A and C is given for the complete set of tests. For this group no attempt will be made to analyze results on the individual questions of the test.

The means and standard deviations have been calculated for each test and any differences between those attained by the two schools have been tested for statistical significance. The results are discussed following Table XIX.

TABLE XIX

SCHOOLS A AND C ON TESTS I TO IV

School A

Pupil	I	II	III A	III I	III S	IV-I C	IV-I G	IV-I W	IV-II CC	IV-II RC	IV-II IR
Possible	16	79	7	25	25	17	-	-	12	23	-
62	16	79	6	25	24	14	3	0	10	14	2
68	15	74	7	25	25	15	2	0	10	7	7
73	12	61	5	24	20	12	5	0	6	12	8
78	12	66	7	25	24	12	4	1	7	11	3
83	13	68	4	21	12	14	1	2	6	6	9
64	11	73	7	25	25	9	6	2	7	11	6
74	10	72	4	25	17	13	3	1	7	12	3
75	14	67	5	24	19	8	6	3	6	10	0
77	16	74	6	25	22	13	2	2	9	15	7
65	7	70	1	18	11	15	1	1	9	7	1
67	10	59	1	21	8	10	6	1	7	6	14
71	16	71	7	25	23	13	3	1	10	11	5
81	16	71	7	25	24	11	5	1	7	11	5
80	14	69	7	25	22	12	4	1	8	11	5
76	12	59	6	24	19	10	6	1	8	12	4
70	10	76	2	25	16	10	6	0	9	6	5
63	9	72	7	23	22	14	1	2	6	13	5
79	9	57	1	16	10	7	7	3	5	4	11
72	11	65	5	18	11	12	4	1	8	5	5
82	8	55	6	22	9	11	5	1	7	7	5
66	13	63	1	22	12	9	7	1	10	11	3
Av.	12.1	67.7	4.9	23.0	17.9	11.6	4.1	1.2	7.7	9.6	5.4
Std. Dev.	2.7	6.5	2.3	2.7	5.8	2.2	1.9	0.9	1.6	3.1	3.0

School C

Pupil	I	II	III A	III I	III S	IV-I C	IV-I G	IV-I W	IV-II CC	IV-II RC	IV-II IR
Possible	16	79	7	25	25	17	-	-	12	23	-
30	16	72	3	19	11	10	6	1	6	10	24
28	15	73	5	19	16	12	2	3	8	6	4
33	15	70	5	25	20	11	4	2	3	9	18
39	15	69	2	8	3	6	8	3	8	4	2
20	14	65	1	12	7	9	6	2	6	6	6
26	15	74	3	14	10	13	3	1	7	8	11
31	9	70	2	10	8	13	2	2	6	6	12
35	12	46	-	-	-	12	3	2	7	7	3
34	14	76	3	18	11	9	3	5	8	11	6
37	3	21	-	-	-	9	3	5	8	8	10
29	11	54	1	13	4	9	5	3	6	8	7
24	16	74	4	15	13	8	5	1	7	10	3
41	5	48	0	10	6	7	8	2	6	7	11
21	-	49	0	14	2	12	3	2	7	10	14
25	14	44	5	15	11	9	6	2	6	1	0
Av.	12.4	60.3	2.6	14.8	9.4	9.9	4.5	2.4	6.6	7.4	8.7
Std. Dev.	4.0	15.4	1.8	4.4	5.0	2.0	1.9	1.2	1.3	2.5	6.3

A - Answers

I - Interpretation

S - Solution

C - Correct

G - Guess

W - Wrong

CC - Conclusion Correct

RC - Reasons Correct

IR - Irrelevant Reasons

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] — [illegible]

[illegible] — [illegible]

[illegible] — [illegible]

[illegible] — [illegible]

[illegible] — [illegible]

The results of these two Schools are in many respects surprising. A general preview of the relative behaviour of the two schools can be obtained from Chart I Page 53. From Table XIX the following comments can be made:-

Test I.- This is the one test where School A falls below School C (also below all C category schools - see Chart I). The difference between 12.1 correct questions per pupil of School A and 12.4 correct questions for School C is not sufficiently large to be significant. In regard to stability of performance School A is superior (Standard Deviation 2.7 to Standard Deviation 3.0 for School C) but here again the difference is not significant. From this we can conclude that School A does not get its high rating because the students have a highly developed ability to solve everyday problems.

Test II.- School A has a mean score of 67.7 per pupil with Standard Deviation of 6.5 compared to a mean score of 60.3 and a Standard Deviation of 15.4 from School C. Thus School A is superior both in regard to level of score and stability of score. Using the technique outlined in the footnote of Page 76 it is seen that the difference in variances is significant. This factor excludes the applying of the Fisher t-test for it shows the samples to be from different parent populations. However we are justified in saying that the results of the Test show School A superior.

Test III.- The answer sheets on this test were handled as were the answer sheets for the two schools previously examined. The scoring of School A and C may be summarized as follows:-

		Answers	Interpretation	Solution
School A -	Mean	4.9	23.0	17.9
	Std. Dev.	2.3	2.7	5.8
School C -	Mean	2.6	14.8	9.4
	Std. Dev.	1.8	4.4	5.0

In all three scores on this test, School A was superior to School C by amounts large enough to be statistically significant.[1] The high average score of 23 out of 25 for School A on Interpretation is interesting as indicating a highly developed ability to interpret data and transfer it into geometrical terms.

Test IV Part I.- For comparison the results may be tabulated as follows:-

		Correct	Guess	Wrong
School A -	Mean	11.6	4.1	1.2
	Std. Dev.	2.2	1.9	0.9
School C -	Mean	9.9	4.5	2.4
	Std. Dev.	2.0	1.9	1.2

1. The probability of obtaining as great a difference by chance as the difference actually observed is as follows:-

Test III Answers - Probability - less than 0.003
Interpretation - Probability - less than 0.0005
Solution - Probability - less than 0.0005

School A excels School C in the number of statements of this test to which it responded correctly, but the difference is on the border line of significance for it has one chance in fifty of occurring even if these were two samples from the same parent population. There is very little difference in the number of guesses the students of each school made. However, School C has significantly more wrong replies per student than did School A. Considering the three scores to give a composite score on the entire test (e.g. 2R-2W-G) then it might be said that School A is significantly superior to School C. Recalling the introductory remarks concerning the type of instruction given in this school the results should not be surprising.

Test IV Part II.- Summarizing, the results are as follows:-

		Conclusions Correct	Reasons Correct	Irrelevant Reasons
School A	Mean	7.7	9.6	5.4
	Std. DEv.	1.6	3.1	3.0
School C	Mean	6.6	7.4	8.7
	Std. Dev.	1.3	2.5	6.3

With regard to the number of conclusions correct and the number of reaons correct School A is superior to School C but the differences are not highly significant.[1] However the number of irrelevant reasons inserted by School C was much greater than the number inserted by School A.[2]

1. Probability of 0.02 by chance
2. Both difference of means and difference of variances are significant.

<u>General Conclusions from the Comparative Study of the Two Schools A and C.-</u> The superiority of School A in comparison with School C seems to be of a different sort to the superiority of School B over School D. The superiority of School A shows up most significantly in the general type of question relating to principles of logical reasoning. School B, in contrast, is superior in the matter of getting the answer to some actual problem. What conclusions can be drawn from these facts? Do they fall within the range of the hypothesis formulated on the basis of the first comparison.

School A students have been taught extensively in regard to the application of, and the generalizations concerning the principles of logic, both in the special field of geometry and also in other subject matter fields. This instruction speaks for itself in the higher scores on Test IV Part I and the more judicious selection of reasons and conclusions in Part II of the same test. But this knowledge of logic has not developed to the stage where it enables the students in this School to score significantly higher on Test I. This seems to support the hypothesis that a knowledge of principles alone, whether of geometry or of logic, is not always accompanied by an ability to apply the knowledge.

Now that we have complete results of School A we can

answer the question raised on Page 1o4, "Is the teacher sacrificing something to achieve his results on principles of logical reasoning?" Judging by the scores on Test IV we can say his students have learned something about the process of reasoning. To do this other aspects have not been neglected. The pupils of School A score as well in other tests as do students of other schools. Teachers should find encouragement in these results. Students can be brought to a given level of achievement on mechanical geometry problems and also be given additional experience in reasoning techniques which might be useful in other fields. Nor need School A rest in a feeling of having achieved the ultimate. There is still room for improvement. Should not the knowledge of reasoning be able to make possible higher score on geometry problems? That is the pupils still need further experience in the application of their knowledge to practical problems such as Test I uses.

CHAPTER VI

PUPIL SUCCESS ON THE INDIVIDUAL QUESTIONS OF THE TESTS

Purpose of the study of the individual questions

The study of the questions individually and the relative success of the students on the questions was undertaken with a view to making the group of tests serve in a diagnostic capacity.[1] If it can be found what type of question offers to the average student the greatest difficulty, then it would be possible for teachers to spend more time on this type of question , if it is felt that the accomplishment of a solution is of sufficient desirability to warrant the time necessary to master the difficulty of the question . The study might also reveal where the difficulties lay: and, in the light of the discoveries of the earlier sections of the study, attempts might be made to remove the cause of the difficulty.

General Method of Attack

Only the results of the four "Type" schools were used, a procedure which is justifiable if the schools have been accepted as typical. Since no attempt is here being made to compare the schools, no attempt is made to keep the schools separate. Rather the results of all the students who are actually taking the geometry course are used to give a composite picture.

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1. See Chapter I Page 5.

Test I

Seventy-three students from Schools I, IV, VII and X worked on Test I. The following table lists the questions in order of increasing difficulty from the point of view of these students. Also included is the number of students out of the seventy-three who got each question correct.

TABLE XX

TOTAL SCORES ON TEST I

Rank	Question	Number getting question correct
1	2	70
2	13	67
3	3	66
4	6	64
5	8	61
6	12	58
7	11	56
8	7	54
9	16	53
10	9	51
11	1	51
12	5	50
13	10	50
14	15	50
15	14	49
16	4	37

II

Those questions on which the students get high scores are of interest only inasmuch as teachers should know that the students have reached near perfection on this variety, and that further drill is not only unnecessary but also, no doubt, is discouraging to the student. In this group of questions might be placed the following:-

Question 2 - based on two distinct similar triangles.

Question 13 - dealing with the radius bisecting the perpendicular chord.

Question 3 - the Pythagorean theorem to find the hypotenuse of a right triangle.

Question 6 - the perpendicular to the hypotenuse of a right isosceles triangle makes equal angles.

Question 8 - relation of the size of angles in an isosceles triangle.

The questions in which the students have the greatest difficulty should perhaps be given more detailed study. Although there are no very low scores when the results are expressed in terms of per cent, still a score of 50 or lower is low enough to warrant examination. The questions and the principles involved are the following:-

Question 4 - Calls for the determination of the equal sides of a right isosceles triangle when the hypotenuse is given. It is interesting to note that this is the same situation that is met in Question I of Test III.#

See comments pages 120 and 121.

Questions 14 and 15 - Given a diameter perpendicular to a chord, and a second chord making a given angle with the first chord; the student is required to find the distance of the first chord from the centre of the circle, and also the angle between the diameter and the line joining the end of the first chord to the centre of the circle.

Question 10 - A compound figure involving both parallel and perpendicular lines. In this case it is likely that the difficulty lies in the involved nature of the figure.

Question 5 - This question involves the same situation as number 4, namely the right isosceles triangle but this time the student is to find the altitude.

No ostensible differences can be found to exist between the problems offering difficulty to students and the problems which the students handle readily. In fact we find in this test the problems involving the isosceles right triangle have a large number of errors while in Test III the problems involving the same principle have fewer errors than any others. The context in which the principle is applied, not the principle itself, seems to determine the difficulty pupils experience in solving the question.

TEST II

There are too many questions in this test to arrange them singly in order of difficulty, so they were arranged in groups according to the number of errors students made on each. It was found that the number of students getting a given question correct varied from 30 to 90. This range was broken up into ten groups each containing a range of six. The questions falling into each group are listed in Table XXI.

TABLE XXI

TOTAL SCORES ON THE QUESTIONS OF TEST II

Rank	No. Correct Answers.	Questions	Total
I	84 - 90	1 2 4 8 9 10 18 19	8
II	78 - 83	3 14 15 28 32 33 49 55 56 66	10
III	72 - 77	5 11 13 20 21 29 30 31 34 48 67 70	12
IV	66 - 71	12 16 17 22 42 43 44 45 58 61 77	11
V	60 - 65	6 41 50 52 54 60 63 65 68 74 75 76 78	13
VI	54 - 59	7 23 24 26 35 46 47 51 53 62 69	11
VII	48 - 53	25 27 37 57 72 73	6
VIII	42 - 47	36 40 64	3
IX	36 - 41	59 71	2
X	30 - 35	38 39 79	3

On the last three groups there were less than half the students who obtained correct replies. (Actually 90 students from these schools answered this test. Only on Question 1 did all 90 give correct response.) The questions of greatest difficulty will be examined in detail.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible] – [illegible]	[illegible]
[illegible]	[illegible]	[illegible] – [illegible]	[illegible]
[illegible]	[illegible]	[illegible] – [illegible]	[illegible]
[illegible]	[illegible]	[illegible] – [illegible]	[illegible]
[illegible]	[illegible]	[illegible] – [illegible]	[illegible]
[illegible]	[illegible]	[illegible] – [illegible]	[illegible]
[illegible]	[illegible]	[illegible] – [illegible]	[illegible]
[illegible]	[illegible]	[illegible] – [illegible]	[illegible]
[illegible]	[illegible]	[illegible] – [illegible]	[illegible]
[illegible]	[illegible]	[illegible] – [illegible]	[illegible]

[illegible]

Question 79 - Only 30 pupils replied correctly. The question required only the inserting of the number of the proposition that justified saying that since two halves of a triangle are congruent therefore two sides are equal and the larger triangle is an isosceles triangle. (Note that this again deals with an isosceles triangle, although it is not right angled. Still the students seem to have trouble with the idea.)

Questions 36, 38, 39 and 40 fall into the groups offering great difficulty. (The other blanks of the exercise - 35, 37 - are also comparatively low in the distribution.) This exercise involves an isosceles triangle where the two equal sides are longer than the third side. From one of these longer sides a length is cut off equal to the shorter side such that a second isosceles triangle is formed inside the first. The student is required to prove the two triangles are similar and that hence a certain proportion exists between the sides. It was the recognizing of the corresponding parts of the inner triangle that seemed to offer the greatest difficulty, for blanks 36, 38, 39 and 40 all deal with the inner triangle.

Questions 59 and 64 deal with an equilateral triangle with side of given length. A median is drawn. Blank 59 asks for the length of this median while blank 64 required the reason which explains why the median makes an angle of thirty degrees with the sides. These particular blanks are preceded by a skeleton solution which indicates that the two triangles formed by the median should be shown to

[illegible]

[illegible]

[illegible]

be congruent right triangles. The difficulties encountered by the majority of the students at this stage seem to indicate that they are not capable of applying the properties of the right triangle in situations which have been complicated by lines other than the three sides of the triangle.

Question 71 called for exactly the same reason as did blank 69. It might have been the feeling on the part of the student that such an immediate repetition must be wrong that caused the high frequency of error at this stage. The idea itself was not difficult, involving only the proposition which states that perpendicular lines make angles of 90°.

The results on this test indicate that greatest pupil difficulty occurs when a geometric figure with which they are already familiar is placed in unusual surroundings. It appears that a knowledge of the properties of the simple figure do not extend or transfer to the more complicated figure. We saw this to be the case for both isosceles and equilateral triangles. The answers on the earlier questions of Test I show that the students have, in general, mastered the principles concerning the common geometric figures when they appear in their simplest and most apparent forms, but the same principles are not conquered when they appear in more complex situations.

Test III

From the four typical schools the results of 73 students are available for this test. The number getting each of the questions of Test III correct is given in the following table. The questions are arranged so that those on which the pupils performed best occur first.

TABLE XXII

TOTAL SCORE ON THE QUESTIONS OF TEST III

Question	Number getting it correct
Question I Part II	51
Part I	50
Question IV	34
Question II	32
Question V Part I	32
Question III	26
Question V Part II	20

Question I in geometrical terms is nothing more than the finding of the altitude and the side of a right isosceles triangle. We saw in previous questions that this situation offered difficulties to the average student. In this test however the students do better on this item than on any of the others of the test. How can this apparent inconsistency be explained? In the previous tests where the student seemed to have difficulty with the right isosceles triangle it was in comparison with other geometric figures which of necessity in a first course of high school Geometry

had to be kept simple. In this Test all the geometrical principles are concealed, to a certain degree, behind a real life problem. Thus we see while complex figures involving the right isosceles triangle do in some cases appear difficult, still in general this figure is better mastered for use than are many of the other geometrical concepts. This must inevitably lead to the conclusion that these other concepts would offer great difficulty if called for in complex figures or where the geometric nature of the problem is not immediately apparent.

Questions IV and II bear out this statement for they are comparatively well handled. On examination it is seen that the geometry figures involved are simple, one being a single isosceles triangle, the other a circle with a chord and a perpendicular to this chord.

The difficulty with Questions III and V seems to lie in the complexity of the figure for the concepts themselves are basic and simple. Question III involves only parallelism, perpendicularity and the sum of the angles in the triangle. Question V is nothing more than a rectangle inside a right isosceles triangle. Thus while a right triangle in Question I is well handled the same concept is poorly handled when the geometric figure is more complex in Question V.

Test IV Part I

As has been described previously this part of the test gives pairs of statements concerning both Principles and Fallacies of reasoning applicable both in geometry and in other subject matter fields. Table XXIII merely lists those pairs and the number of students from the four typical schools who reply correctly to each. The information was secured from Table III. The pairs are arranged so that those offering the least difficulty are first.

TABLE XXIII

TOTAL SCORES ON THE STATEMENTS OF TEST IV PART I

Rank	Pair	Number getting it correct
1	5 & 33	67
2	1 & 21	66
3	20 & 28	59
4	18 & 30	57
5	7 & 29	56
6	14 & 26	55
7	4 & 13	54
8	10 & 31	54
9	3 & 32	50
10	2 & 24	48
11	11 & 22	42
12	6 & 17	41
13	8 & 16	39
14	12 & 19	32
15	25 & 34	25
16	9 & 23	18
17	15 & 27	13

With a total of 73 students' results included, the statements which fewer than 40 pupils marked correctly were examined with several questions in mind. In the first place the statement itself would be examined to see that it is correct, clear and not ambiquous in its wording.

Then it would be wise to ask ourselves the question, "Is it worth while that High School students know this principle?" If we answer this in the affirmative then, and then only, should we become concerned with the poor showing that the pupils have made dealing with the statment pair. The reason for the pairing of the statement should perhaps be repeated at this stage. It was felt that it would eliminate, to a large extent, the possibility of a student obtaining high score because he had memorized a number of statements.

Pair 15 and 27, which gave greatest trouble expects the student to realize that in every argument there must be some starting point, some assumption or unproved proposition, and that the existence of this does not invalidate the entire argument. This principle, since basic to all scientific thought, is very important. Certainly also it is a valuable item of knowledge to students who are to be expected to examine critically the statements of organizers, politicians, insurance salesmen and promoters who all claim there isn't a doubt concerning the statements they make. True enough not a very large proportion of our adult population apply the principle but this makes it no less valuable. Obviously, from the results on each pair is a compound even with two possibilities to each, a chance distribution for the 73 students would results in 18 correct, 56 guesses and 18 wrong. Only 13 correct responses on this pair indicates an entire absence of understanding.

Pair 9 and 23 is based on the idea that a correct conclusion can be reached without every word in the argument being formally defined, if the people concerned are in agreement concerning its meaning. This again obtains a score which might be accountable for by chance.

Pair 25 and 34 relates to one of the commonest fallacies of both geometry and everyday argument. This fallacy is that of using in the argument a statement that is equivalent to the assuming of the conclusion to be proved. It is the type of thing that the lawyer is doing when he refers to the man on trial for murder as "The Murderer". Geometry certainly offers many chances to bring to the attention of the student this fallacy. Situations which are not colored with numerous prejudices are available more readily in the field of Geometry than in any other field.where the student has a class-room teacher. In view of this, the response is very disappointing.

The Pair 12 and 19 deals with the revelance of the reasons in the supporting of an argument. To be useful in supporting an argument, a principle in addition to being true, must also relate to the argument. Certainly students already recognize this concept to a certain extent but still they need specific training and drill to make them clearly conscious of its wider applications. The Pair 8 and 16 deals with the extent to which a dictionary settles disputes concerning the meaning of words. An examination of the statement of the above two pairs of principles in the test itself

showed that the wording was not altogether without ambiquity. The low score in part might be due to this. Certainly revision is necessary before the test is used again.

Included in the test are many concepts which every student of the modern world should know and which can be taught through the use of geometry. Although in certain cases an attempt to keep the statements simple resulted in possible ambiquity still the results of Part I of Test IV indicate that many of the concepts involved have not been mastered to any degree of perfection by Geometry students. However the high scores made on some of the pairs of statements show that a certain amount of progress is being made. This should be a challenge to every teacher when he realizes that something can be done and needs to be done.

The results of a group of University students corroborate the results obtained from the high schools.[1] The order of difficulty was approximately the same - pair 15 and 27 lowest score and then pair 9 and 23. There was the same wide range of scores. Out of 57 students only 11 were right on pair 15 and 27; 56 were right on pair 1 and 21. Thus we see there is still need of emphasis on this phase at the University level.

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1. Fifty Seven First Year Mathematics Students, Faculty of Arts, University of Alberta. 1942-3

Test IV Part II

Several situations formed the basis of this part of Test IV. The student was expected to choose the conclusion justified by the situation and then to support his choice by reference to one or more of the principles of reasoning that formed the basis of Part I of the test. Of the 73 students from the four typical schools the number getting each conclusion correct is recorded. The blanks for conclusions are then ordered according to the total number of students getting them correct and the results entered in Table XXIV.

TABLE XXIV

TOTALS ON CONCLUSIONS OF TEST IV PART II

Rank	1	2	3	4	5	6	7	8	9	10	11	12
Blank	8	3	20	19	1	6	16	2	14	10	12	5
Total	66	66	61	58	56	49	46	42	41	36	33	11

Blank 8 which is well handled is based on the idea that a correct conclusion can be arrived at without a formal definition, if all the parties concerned are in argument concerning the meaning of the words involved. It is particularly interesting to note that this idea as Pair 9 and 32 was the second most poorly handled in Part I of this Test. The better showing in this context might indicate that many students realize the truth of the statement but do not recognize it when formally worded.

Blank 5, which is only recognized correctly by 11 out of the 73 students, also deals with definition. It involves the recognition of the necessity of agreement concerning every word of a definition before the definition is of any use in the settling of an argument. What can we conclude from the irregular performance with regard to principles dealing with definitions? It seems that under favourable circumstances - as Blank 8 - pupils can recognize the role of definition, but if the situation is more complicated, or the wording less familiar then the students are no longer cognizant of the true nature of the principles involved.

In replying incorrectly to Blank 12 the students showed they had not been trained to recognize the fallacy of an argument which both assumed the converse of a theorem and assumed in the proof that which was to be proved. We saw on Part I of this test that the pupils did not understand the latter of these principles. - Pair 25 and 34 - Here is further evidence.

The situation on which Blank 10 is based involves a construction which assumes what is to be proven. The student was asked whether he would accept a proof based on this construction. Table XXIV shows that only about half the students recognized the fallacy.

Blank 14 expects the student to recognize the fallacy in an argument which assumes the converse of a theorem- or more accurately uses a theorem where its converse is needed.

The poor results here, along with the results on Blanks 10 and 12 indicate that even simple fallacies of reasoning such as arguing in a circle or assuming a converse are not at all uniformly well handled by high school students.

Following this analysis of conclusions of Test IV Part II the use of supporting reasons was considered. Some of the reasons could be used to support several conclusions. The total number of times the reason was used correctly by all the students was divided by the number of places in the test the reason could be correctly used. The result was the mean number of times the reason was used correctly to support each conclusion to which it was applicable. The total number of times the reason was irrelevantly used was also recorded and this was subtracted from the total correct uses with a view to giving for each reason some measure of accuracy of use. These figures are included in Table XXV.

TABLE XXV

ACCURACY OF USE OF REASONS IN PART II OF TEST IV

R	The Reason	No	Mn	Tl	Ir	Sc.
I	Importance of defining all words used in an argument.	3	19	58	10	48
II	Every word in a definition must be agreed upon.	2	33	66	37	-4
III	Formal definition not required where there is agreement about meaning of word.	1	61	61	51	10
IV	Changing meaning of a word in a definition changes definition.	1	24	24	21	3
V	Different meaning to word in a definition results in different conclusions.	1	13	13	48	-35
VI	A word may be correctly defined in several ways.	1.	24	24	21	3
VII	Change in definition results in different conclusion.	2	14	27	28	-1
VIII	Converse of a theorem is not always true.	3	17	52	22	30
IX	It is not correct to argue by assuming what is to be proven.	3	16	48	50	-2
X	Constructions should not rest upon or imply what is to be proven.	2	29	58	32	26
XI	All arguments must start with assumptions.	0	0	0	91	-91
XII	All possibilities must be considered in an indirect argument.	1	30	30	42	-8
XIII	Can only apply a theorem when conditions of theorem fulfilled.	2	39	78	43	35

R. Number assigned to Statement in list on Page 32.
No. Number of times reason can be correctly used on the Part.
Mn. Mean number of times reason used correctly.
Tl. Total number of times reason used.
Ir. Number of times reason used irrelevantly.
Sc. Score for reason - Correct uses minus irrelevant uses.

We see that statement III dealing with the use of undefined but mutually agreed upon words is used correctly in a large number of cases. However it has a large number of irrelevant uses indicating that students do not completely understand the implications of the statement. Actually there were a large number of inconsistent answers regarding this statement in Part I of Test IV. Since all the statements to be used in Part II were correct the student had merely to recognize that the point at issue was a matter of definition in order to be able to pick out the correct reason. Also in the field of definition is Statement I which seems to be very well handled. The wording of this statement is quite simple, a fact which might explain the better results.[1] Statement I concerned with the importance of definition is relatively well handled. The score is higher and the statement has fewer irrelevant uses than for any other statement.

Also well handled is Statement XIII. This theorem deals with the necessity of a special case fulfilling the conditions of a theorem before that theorem can be applied to the case.

The fallacies of assuming a conclusion that is to be proven, or using the converse of a theorem seem to be well understood in theory for Statements VIII and X give high scores.

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1. See comments on Blank 5 Table XXIV and Page 127.

The fact that every deduction is based on some unproved proposition as given in Statement XI seems to come in for a large amount of misuse.

The low results on Statement V show that in general students have difficulty in recognizing the effect on a conclusion that results when the meaning of one of the words in a definition is changed. The necessarily longer, more involved wording of this statement might in part account for the difficulty experienced.

The figures shown in the last two tables were not included here with the purpose of revealing once and forever the kind of error that first-course geometry students make in the sphere of logic. Rather, the intention was to show what is being done in this field and where there is room for further work to be done. It can be safely said that the students' knowledge of these principles of logic is still of the theoretical type that is not yet too readily applicable to the every day situation. As long as it remains of this type and instruction has not yet achieved its maximum possibilities.

CHAPTER VII

GENERAL CONCLUSIONS OF THE STUDY

To draw general conclusions from a study of this type where most of the work is detailed and specific is difficult, for in such an analysis slight changes in the wording of questions may have as large effect as changes in the actual nature of the problems. This fact may account for any apparent inconsistencies between the various parts of the study. Many conclusions have been stated in the body of the study as results seemed to warrant. These conclusions which have already been stated may be grouped into two sections: those derived from the comparative study of the two pairs of schools and those derived from the study of the relative difficulty of the individual questions of each test. The summarizing of the conclusions here will be directed to the answering of some of the numerous questions that have been raised from time to time throughout the study.

In order that the statement of results might be clear the usage of certain phrases should possibly be mentioned again. When reference is made to a students' theoretical knowledge or his knowledge of theoretical principles, then we mean that which is measured by his score on actual tests of the older classroom type where he is questioned on general principles, either of geometry or of logic, which are not

required to be applied to everyday problems necessitating clear and original thinking on the part of the student. Test IV Part I checked this type of knowledge in the field of logic; Test II and to a lesser extent Test I did the same thing in the sphere of geometry. The "practical knowledge" of a student has been measured by Tests II[I] and Test IV Part II. The problems or situations in these tests were commonplace and everyday. Consequently it was felt that the ability to apply the principles mentioned above could appropriately be termed "practical".

Conclusions from the Comparative Study of Schools.

One truth is apparent from the results that have been examined. This truth is that there is not always a relation between pupil's knowledge of theoretical principles of reasoning or of geometry and his ability to actually solve problems. The results time and time again brought out this fact.

In comparison of School B and School D there was no significant difference in the knowledge of the two schools on theoretical concepts either of geometry or of logic, but still School B was much superior when it came to the actual solving of problems. This was evidenced by the many cases in which a student obtained the correct answer in Test II

where an outline of solution was supplied, but did not get the equivalent Test I problem where he had to go ahead and supply his own solution. Also in Test III we saw that there was little difference between the two schools in the matter of interpretation, but still School B did noticeably better in regard to solution and hence also in the matter of correct answers.

The comparison of Schools A and C illustrated this divorce between theoretical knowledge and practical knowledge in a different manner. School A did not do much better than its mate in regard to the solution of actual problems even though it was noticeably superior in regard to theoretical knowledge of concepts of both logic and geometry. The superior theoretical knowledge of School A shown in Test IV Part I did not automatically result in superior practical knowledge. When the teacher questionnaire of School A was examined it was pointed out that this teacher has a particularly keen interest in the logical aspects of geometry. Consequently it is reasonable to assume that he would stress these aspects in his teaching. Hence the high score of his school on Test IV Part I. The point to notice here is that, even with this superior training and knowledge in theoretical principles, his school did not score significantly superior to the matched school.

The pedagogical implications of this is that to instruct a class in the geometric theorems of the type laid down by Euclid and to elaborate on the abstract principles of reasoning is not sufficient if utility is to be an outcome. Knowledge of theoretical principles alone is not sufficient to assure that the student can either solve mathematical problems where his geometric knowledge is sufficient or that he will extend his reasoning techniques to other types of subject matter. It would appear that it is necessary to instruction in the above and in addition give the pupil actual experience in the application of the knowledge to situations in which he finds himself. These findings are in accordance with the results of Rolland Smith referred to on Page 2 of this study. To the question raised on Page 4 concerning the relation between a student's ability to solve actual specific problems and his mastery of the logic of the problems, the results would answer that there is no clear cut relation discernable.

Conclusions from the Study of the Individual Problems of the Tests.

Early in the study[1] the hope was expressed that the study would yield specific information concerning the type of problem offering greatest difficulty. However no general statement as to the type of problem follows from the study of the various items of the tests. Rather we are led to the opinion that it is the context in which a principle

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1. Page 4

is located that determines its difficulty to the average pupil. Thus if a problem involving the right isosceles triangle appears in a simple set-up, then the student applies his knowledge with a degree of facility. But if the same knowledge is called for in a problem where there are more factors involved, where the figure is more complex, or where the information is not given in the exact form in which it is required, then the pupil response will indicate that the problem is too difficult for the majority of the students.

This generalization has practically the same pedagogical implication that was made above; namely that some of the instruction time - even a large proportion - should be spent in the actual solving of problems. It is to be noted that none of the schools examined did poorly on the theoretical tests and then turned around and did well on the practical tests. As a result we can still say that the theory is necessary but this same theory must be applied.

In the outlining of the purpose of the investigation the question, "Can geometry be given direct and practical usefulness and at the same time give the pupil certain ideas about the nature of proof?" was asked. Now we should be in a position to answer this question. Geometry is practical and useful if the student can use it to solve the

everyday type of problem such as was given to him in Test III. He has gained "certain ideas of the nature of proof" if he can score well on Test IV and on Test II. Have any Schools done superior work in both fields? School I of Category A did especially well in regard to the ideas concerning the nature of proof and at the same time did as well as most of the schools in actual practical problems. Then in this school the teaching of geometry has accomplished this dual role of being practical and at the same time giving some notion of the nature of proof. What has been done once can be done again. This then is the challenge that the study offers to every teacher.

APPENDIX I

TEST V

TEST OF CERTAIN ASPECT OF LOGICAL REASONING

Devised by S.C.Clarke.

TEST V. TEST OF CERTAIN ASPECTS OF LOGICAL REASONING

neral Directions:

You have been given an answer sheet to go with this test. Fill in e blank spaces at the top of your answer sheet.

No time limit has been set for this test. If possible, sufficient me should be allowed for the completion of all the problems. About xty minutes will probably be sufficient.

Do not put any marks on this test booklet.

You will find in this test booklet a number of problems, each of ich is a situation from everyday life. You are asked to consider the asoning process used in each situation.

AN EXPLANATION OF SOME TERMS USED IN THIS TEST

direct Argument:

In an indirect argument a number of possible causes or conditions are nsidered. All are eliminated except one, which then becomes the nclusion.

Example: "According to Holmes, there were only four ways of entering e room--through the door, through the window, down the chimney, or rough the trap-door in the ceiling. The thief did not enter through e door, nor through the window, and the chimney was too small for him enter that way, so he must have entered by the trap-door."

In this example of indirect argument the underlined statement is the nclusion.

verse:

A proposition which interchanges the subject and predicate of another position.

Example: Proposition:--All figures with four right angles are rectangles.
Converse:-----All rectangles are figures with four right angles.

SAMPLE PROBLEM

The members of a high school class in Social Studies were discussing seizure of parts of China by Japan, and all agreed that a nation is tified in demanding the return of former territory. As Henry put it, ce an area has belonged to a country, it should remain a possession that country. In too many cases unjust wars by a powerful aggressor e stripped a country of her rightful territory".

When John pointed out that since Quebec province once belonged to nce, France could rightfully demand its return as a colony, everybody ghed.

irections:

Assume that all the above statements are true, except the ne underlined. Use only the data given in the problem. According o the principles of logical reasoning with which you are familiar, ndicate what you think of the underlined statement by choosing one of he following conclusions.

nclusions:

John proved that France could rightfully demand the return of Quebec province as a colony.

John did not prove that France could rightfully demand the return of Quebec province as a colony.

More information is needed to decide whether or not John proved that France could rightfully demand the return of Quebec province as a colony.

ote:

Since you are to assume that all the given statements (except the ie underlined) are true, you must accept as true that all the class greed that a nation is justified in demanding the return of former erritory. Further, you must also accept as true that Quebec province ice belonged to France. Your conclusion about John's reasoning should conclusion (A), above.

Henry's statement, and the fact that everybody laughed, are addi- onal facts not required in determining which conclusion to choose.

rections:

Choose the reasons which support or explain the logic of ur conclusion.

sons:

The word "territory" must be defined before we can decide whether or not John proved his point.

John deserved to be laughed at for such a silly statement.

If we accept certain assumptions, then to be logical we must accept the conclusion which follows from them.

John was trying to stir up racial prejudice in Canada.

Canada was at one time a possession of France.

Britain gained Canada as a result of war.

We must admit that France could rightfully ask for the return of Quebec province as a colony if we agreed that Quebec was formerly French territory, and that a nation is justified in demanding the return of former territory.

No schoolboy such as John can argue about questions which baffle our great statesmen.

Some of the students may not have agreed that a nation is justified in demanding the return of former territory.

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ote:

The important statements are those which are related to the ·inciple that if we accept certain premises we must accept the logical onclusion which follows from these premises.

The statements to be chosen are numbers 3 and 7.

Where reasons add to or contradict the data of the problem, re- mber that you are to use only the data given in the problem and the inciples of logical reasoning with which you are familiar, in termining the conclusion. Thus reason #9, which denies what you are ld to accept, must be ignored.

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ording your answer.

	Conclusion			Reasons											
	A	B	C	1	2	3	4	5	6	7	8	9	10	11	12
Sample Problem	X					X				X					
Problem 1.															

Fill in the conclusion you choose for problem 1 and the reasons you select, just as the sample above is filled in. Place your answers on the answer sheet.

Do not put any marks on this test booklet.

Problem 1.

"The members of a science class, as a result of an argument, decided that they would disregard any conclusions they held at the moment and experimentally determine whether sound required a medium such as air to travel in. An electric bell was set ringing and together with a pressure gauge, was placed in a bell jar. The bell jar was then connected to a rubber hose which led to a fixed air pump in the basement. The gauge reading gradually declined to 2 pounds per square inch. At that time the ringing of the bell sounded much fainter than before.

Albert, a member of the class, when considering the results of this experiment, argued that he knew <u>that air had been removed from the bell jar</u> because the ringing of the bell was fainter.

Directions: Use only the data given above. Assume that the above statements, except the one underlined, are true. Indicate what you think about the underlined statement by choosing one of the following conclusions according to the principles of logical reasoning with which you are familiar.

CONCLUSIONS:

A. Albert proved that air had been removed from the bell jar.
B. Albert did not prove that air had been removed from the bell jar.
C. More information is needed to decide whether or not Albert proved that air had been removed from the bell jar.

Directions: Choose the reasons which support or explain the logic of your conclusion.

REASONS:

- A changed definition will lead to a changed conclusion even though the argument from each is logical.
- Only a fool would say that air had been removed from the bell jar so Albert couldn't have proved that it had.
- Since the class agreed to disregard any conclusions they held at the moment, Albert is not justified in arguing that air had been removed from the bell jar because the bell sounded fainter.
- The pressure gauge registered 2 pounds per square inch so the air had been pumped from the jar.
- The word "fainter" must be defined before we can decide whether or not Albert proved that air had been removed from the jar.
- It is false reasoning if you use the statement you set out to prove as an argument in the proof.
- We must accept the conclusion which follows logically from an assumption which we have already accepted.
- Albert wanted the bell to ring fainter so that he could show that air had been removed from the bell jar.
- Albert gave a good reason for saying that air had been removed from the bell jar.
- One cannot tell from the readings of the pressure gauge whether air had been removed from the bell jar or not.
- Albert proved his point because the school janitor, on hearing of the discussion, stated that he was sure that sound required a medium to travel in.
- Since it had not yet been determined that sound requires a medium to travel in, Albert is assuming the conclusion that the class set out to prove.

roblem 2:

"A small western town has a special license for restaurants. Mr. orquist, a merchant of this town, is the owner of a small general store hich sells, among other things, cold meats, bread, pickles, and the like. oor farmers often come in, buy sausages and bread, cook the sausages on ne open-top heater in the store, and have lunch there. Mr. Norquist has table with a bench at which they eat. Sometimes Mr. Norquist helps them ook. They use a frying pan supplied by Mr. Norquist.

The mayor of the town stated that according to the town's by-laws, r. Norquist would have to obtain the special restaurant license. Mr. orquist replied that he didn't keep a restaurant".

Directions: Accept the above statements as true. Use only these data. According to the principles of logical reasoning with which you are familiar, select the conclusion which you think applies.

ONCLUSIONS:

Mr. Norquist should not have to obtain the special restaurant license.
Mr. Norquist should have to obtain the special restaurant license.
More information is needed to decide whether or not Mr. Norquist should have to obtain the special restaurant license.

Directions: Choose the reasons which support or explain the logic of your conclusion.

ASONS:

Mr. Norquist wants to get additional business by underhand means.
A changed definition will lead to a changed conclusion even though the argument from each definition is logical.
To make a sound indirect argument, one must consider all the possibilities.
Mr. Norquist was helping the town by helping the poor farmers so he should not have to pay for the special license.
Anyone who claims Mr. Norquist didn't keep a restaurant is just plain stupid.
We must know whether a place where people buy food, have it cooked, then eat it, is legally a restaurant.
A place where people drink beer is a beer parlor so a place where people eat meals is a restaurant.
The Principal of the school said that Mr. Norquist was operating a restaurant.
There is sufficient evidence to show that Mr. Norquist was operating a restaurant.
Since Mr. Norquist always helped cook, he was really selling cooked food.
One is not justified in using the statement one set out to prove as part of the argument.
If the mayor is going to argue that way, he must show that Mr. Norquist does not operate a grocery store, a hardware store, a confectionery (and so on for all kinds of business), so he must be operating a restaurant.

roblem 3.

"A high school class in English had a debate 'Resolved that the use cosmetics should be freely permitted in school'. At noon, Hazel and ny continued the argument as follows:

NY:--"You girls just want to 'high hat' the high school boys. You doll up so that the older boys will take you out".

ZEL:-"'Taint so. Using cosmetics improves a girl's appearance and makes her attractive."

NY:--"What do you mean by 'girl'?"

ZEL:-"A high school girl. Appearing attractive gives a girl more confidence and so improves her personality."

NY:--"Yes, but a painted doll looks terrible. You have no discretion. Girls shouldn't be allowed to use cosmetics in school".

ZEL:-"I have too got discretion! Anyway, the use of cosmetics improves a girl's personality, and anything that will improve a girl's personality should be used by girls".

NY:--"I agree with everything you said so far, except that you haven't any discretion".

ZEL:-"Then you must admit that cosmetics should be used by high school girls."

irections: Put yourself in Tony's place. Choose one of the following conclusions.

NCLUSIONS:

To be logical, Tony must admit that cosmetics should be used by high school girls.
Even if he is perfectly logical, Tony still does not have to admit that cosmetics should be used by high school girls.
Even if Tony is perfectly logical, we need more information to enable us to decide whether or not Tony must admit that cosmetics should be used by high school girls.

rections: Choose the reasons which explain or support the logic of your conclusion.

ASONS:

Since Tony agreed to all of Hazel's statements except that she had discretion, he must admit the logical conclusion which follows from these statements.
Cosmetics should have some definition such as "lipstick, rouge, powder, and mascara" before we can decide whether or not Tony must admit Hazel's point.
Tony was insulting and talked like a smart alec instead of arguing, so he should admit Hazel's point.
It is quite true that painted dolls look terrible and that most high school girls have no discretion, so Tony need not admit Hazel's point.
When one changes a definition, the conclusion is changed even though the argument used is still logical.
Bronx High, the best high school in the City of New York, encourages girl students to use cosmetics.
If one accepts certain assumptions, even though the conclusion which logically follows is not necessarily true, it should be accepted.
Just as a brightly painted barn doesn't look attractive, a girl with crimson lips doesn't, so Tony should not have to admit Hazel's point.
Hazel must be the kind who wants to flirt with the high school boys and that is why she thinks high school girls should use cosmetics.
We need to know whether Tony's statement that no high school girl ever has any discretion, is correct.
In a logical proof the statement one set out to prove cannot be used as an argument.
Since Tony must admit that cosmetics should be used by high school girls he cannot be a logical arguer.

oblem 4.

A young Nazi prisoner one day had an argument with his guard. Here part of it: "Some of Herr Hitler's conquests have been brilliantly anned. Take the occupation of Norway. The surprise element was fectively used. Look how astonished the stupid English swine were! rwegian sympathizers were carefully employed to the best advantage. Co-dinated sea and air attacks were made with small losses. Anything that brilliantly planned is worthy of admiration. The pictures in the Art llery at the Louvre are famous because they combine a careful and illiant planning with masterly execution. A great and beautiful build-g like the Empire State Building has to be brilliantly planned. You st admit that some of Herr Hitler's conquests are worthy of admiration. il Hitler! Down with Democracy!"

The guard replied that he was a slick talker but that nothing Hitler d deserved admiration.

rections: Accept the prisoner's statements (except the one underlined). Using the above data, choose one of the following conclusions

NCLUSIONS:

The prisoner proved that some of Hitler's conquests are worthy of admiration.

Before we can decide whether or not the prisoner proved his point, more information is needed.

The prisoner did not prove that some of Hitler's conquests are worthy of admiration.

rections: Choose the reasons which support or explain the logic of your conclusion.

SONS:

Nothing such a treacherous, callous brute as Hitler (the murderer of women and children) does, deserves any admiration.

The prisoner and guard must agree on the definition of "brilliantly planned" before we can decide whether or not the prisoner proved his point.

The prisoner wants to destroy democracy and called the English "stupid swine", therefore he could not possibly have proved that some of Hitler's conquests are worthy of admiration.

If one agrees that anything brilliantly planned deserves admiration and that the conquest of Norway was brilliantly planned, the prisoner's conclusion follows.

The prisoner was trying to convert the guard to Nazism so that he could escape.

A changed definition leads to a changed conclusion even though the argument from each is logical.

In such an argument as this we must consider Hitler's conquest of all other countries, not only Norway, before we can be sure of the correctness of the conclusion.

The conclusion is not necessarily true, but if one accepts the premises on which it is based, to be logical one must accept the conclusion.

The famous airman, Col. Charles A. Lindbergh, admires some of Hitler's works.

The prisoner was abusive but he showed that if some of Hitler's conquests have been brilliantly planned and anything brilliantly planned is worthy of admiration, then some of Hitler's conquests are worthy of admiration.

In an indirect argument it is necessary to consider all the possibilities.

Just as you can't make a silk purse out of a sow's ear, so you can't show any qualities worthy of admiration in a man like Hitler.

oblem 5.

A speaker at a business men's banquet told the following story:
"Despite the shortage of man power in vital defence industries, our g cities are still crowded with 'bums' and 'dead-beats'. Just the her day a big strapping fellow with toes out at his boots and a three y's growth of beard, came up to me and pleaded: 'Nickel for a cup of ffee, Mister?'
'Shame on you', I replied, 'Have you no self-respect, to be begging en Canada's war industries need men for the defence of democracy?'
The dirty bum got quite huffy and replied, 'Respectable people want to rk. I want to work. I've looked high and low for work. I'll have u know that I have plenty of self-respect. I'm a respectable man'."

irections: Assume that the above circumstances are true, and that he statements attributed to the 'bum' are true, except the one nderlined. Indicate what you think about the underlined statement y selecting one of these conclusions:

NCLUSIONS:

The 'bum' did not prove that he is a respectable man.
More information is needed to decide whether or not he proved that he is a respectable man.
The 'bum' proved that he is a respectable man.

irections: Choose the reasons which support or explain the logic of your conclusion.

ASONS:

A person with plenty of self-respect must be respectable.
A man claimed that he had looked for work but we must have more information about this because one cannot rely on the statements of a 'bum'.
The conclusion is not necessarily true, but if one accepts the premises on which an argument is based, to be logical he should accept the conclusion which logically follows.
Before we can decide whether or not he proved his point we must have a precise definition of "respectable".
The man probably wasn't a 'bum' but the speaker was a prosperous business man so he tried to picture the man as a 'bum' deliberately so that the speaker himself might appear in a more favourable light.
Even respectable people sometimes can't find work. The poor man might have been starving.
The 'bum' accepted without proof the truth of the converse of the statement, "Respectable people want to work".
If one changes a definition the conclusion which logically follows is also changed.
The chairman of the banquet, at the conclusion of the address, stated that no respectable men were unable to find work nowadays.
The 'bum' assumes the conclusion he set out to prove when he said, "I have plenty of self-respect".
The converse of a true statement is not necessarily true.
In a logical argument one cannot use the conclusion one set out to prove as part of the argument.

oblem 6.

The following item recently appeared in a school paper:

LUSCAR HOCKEY TEAM BEST

"The Inter High School League, made up of Luscar, Bella, Setter, and ayville high school hockey teams, is approaching the play-offs. Because the lack of finance the number of games has been limited. Here is the anding in the games which have been played so far:

Jan. 21	Bella beat Setter	7-2
Feb. 3	Trayville beat Bella	5-1
Feb. 15	Luscar beat Trayville	5-3

One of these teams must be the best. The best team, of course, always ns any well-conducted hockey match. With the able refereeing of Spike Kernan, all the Inter High School League games have been well conducted is year, which is in sharp contrast with last year's showing.

Loyal sportfans who analyze the above figures and arguments will conude with Sportswrite that Luscar team is the best in the Inter High hool League. Luscar should win the Trophy."--by Sportswrite.

rections: You are to assume that all of the above statements, except the ne underlined, are true. On the basis of the above data, indicate what you ink about the underlined statement by choosing one of these conclusions.

NCLUSIONS:

Sportswrite did not prove that Luscar team is the best.

More information is needed to decide whether Sportswirte did or did not prove that Luscar team is the best.

Sportswrite proved that Luscar team is the best.

rections: Choose the reasons which support or explain the logic of your conclusion.

SONS:

Sportswrite assumes that Luscar team is the best and uses that as an argument.

Amateur forecasters about hockey usually need an ice bag to cool them off.

The girl-friend of the Luscar coach gave Sportswrite a hot tip that the team was best.

A changed definition leads to a changed conclusion even though the argument from each definition is logical.

Just because Sportswrite showed that Bella beat Setter, Trayville beat Setter and Luscar beat Trayville, doesn't mean that Luscar is the best team.

In a sound argument one cannot use the conclusion one set out to prove as part of the argument.

We must have a proper definition of the word "best" before we can decide whether or not Sportswrite proved that Luscar team is best.

Since Sportswrite stated that one team must be best, and according to his definition and the scores, showed that Setter team wasn't best (Bella beat it), Bella team wasn't best (Trayville beat it), and Trayville team wasn't best (Luscar beat it); Luscar team must be the best team.

Sportswrite proved that the best team in this year's Inter High School League wins any game it plays.

In an indirect argument we must consider all the possibilities and eliminate all except one, which becomes the conclusion.

Just as the best side wins a war, so the best hockey team wins the game.

If we accept the assumptions on which an argument is based, to be logical we must accept the conclusion which logically follows, even though this conclusion is not necessarily true.

oblem 7.

According to Alberta Law, the only reasons for which a student may excused from attendance at school are:
If he is sick.
If he is over 15 years of age.
If he is at necessary work by permission of the Principal of the school or a Justice of the Peace (for periods of not over six weeks).
If he has passed Grade VIII and further instruction is not offered in his local school.
If the Department of Education is satisfied he is receiving adequate instruction at home.

In September, a farm boy, Kenneth Brown, had missed two weeks of hool. The teacher knew he couldn't be 15 for another half year. He dn't passed Grade VIII. He wasn't receiving any instruction at home-- fact, he was helping with the harvesting. The family was very poor d could not afford to hire any help. The Principal of the school had t given his permission for Kenneth to work at home.

The teacher concluded that his absence from school was not excusable d notified the attendance (truancy) officer.

rections: Assume that the above statements are true. Use only the ta given above. Select the conclusion which you think applies.

NCLUSIONS:
Kenneth's absence from school should be excused.
Kenneth's absence from school should not be excused.
More information is required before we can decide whether or not Kenneth's absence from school should be excused.

rections: Choose the reasons which support or explain the logic of your conclusion.

ASONS:
Kenneth might have been sick and so missed the two weeks of school.
The teacher was picking on Kenneth to report him in the circumstances given.
If one accepts the assumptions on which an argument is based, to be logical he must accept the conclusion which logically follows.
We must have a precise definition of "excusable", "necessary work", and "adequate instruction" before we can decide whether or not he should be excused.
This teacher must have been one of those fussy ones who always stick to technicalities.
A sound indirect argument considers all the possibilities and eliminates all but one.
When one changes a definition the conclusion is changed even though the argument from each definition is logical.
Farm boys should not be allowed to help with harvesting even though the family may be poor, because education is so important.
Since Kenneth's absence was not excusable the teacher was justified in notifying the attendance officer.
When fighting a war every man must do his part, so when the family is fighting a war against poverty, Kenneth must help with the farm work.
Kenneth might be working on permission of a Justice of the Peace.
Using the conclusion one set out to prove as an argument is not sound reasoning.

roblem 8.

A university student was reading a book on "How Men Think" when he as struck by this statement:

"A critical thinker can recognize problems, weigh evidence and draw arranted conclusions and generalizations from the data given".

"Well," he said, "I can recognize problems and weigh evidence. I an also draw warranted conclusions and generalizations from the data iven. Although the author has not defined a critical thinker in this tatement, I believe it is true as far as it goes".

irections: Let us assume that the student could do all these things he aid he could and let us accept in the same way as the student did, the ruth of the above statements about a critical thinker. Indicate what ou think of the underlined statement by choosing one of the following onclusions:

"I conclude", said the student, "That I am a critical thinker".

ONCLUSIONS:

. The student proved that he was a critical thinker.
. The student did not prove that he was a critical thinker.
. More information is required to decide whether he did or did not prove that he was a critical thinker.

rections: Choose the reasons which support or explain the logic of your conclusion.

ASONS:

A straightforward definition of "critical thinker" is required before we can decide whether or not he proved his point.
This student just boasted about himself instead of giving sound arguments so he showed he wasn't a critical thinker.
The book "How Men Think" may not have been written by a reliable author who is an authority on Psychology and Logic.
Without any proof, the student accepted the converse of the statement, "A critical thinker can recognize problems, etc.", as true.
Changing the definition will change the conclusion even though the argument from each definition is logical.
A sound indirect argument considers all the possibilities and eliminates them all except one.
Just as a person who is free, white and 21 can vote, so a person who can recognize problems, weigh evidence, etc., is a critical thinker.
Since the student has eliminated all the possibilities by showing he can recognize problems, weigh evidence, etc., the remaining possibility must be correct and he has proved that he is a critical thinker.
Having accepted the truth of certain statements, to be logical we must accept the conclusion which logically follows even though it is not necessarily true.
Since the student failed to show that he had all the qualities of a critical thinker listed in the quotation given, he did not prove that he is a critical thinker.
The converse of a true statement is not necessarily true.
The student's teacher of English remarked in class just two days before, that this student was a careful, critical thinker.

roblem 9.

"HEAVY SNOWSTORM FLATTENS CROPS"

"Old timers say that never before in their memory has such an early torm occurred. Driven for five hours by a heavy wind, the granular snow lattened all early crops. Farmers report that losses will be heavy". -From Granham Times, August 12.

Conrad Metz, a farmer of Granham area, had his crop insured against ail with the Mutual Hail Insurance Co. Immediately after the storm, he elephoned Granham for a hail insurance adjuster. The local agent report-d, after a time, that the Head Office refused to send out an adjuster ecause there was no hail.

Mr. Metz wrote the following letter to the company:

"I estimate a 50% loss from the hail storm of Aug. 12. The news-apers reported snow; but the local teacher and all my neighbouring armers will testify that the storm was driving hail. A heavy thunder-torm with some rain preceded the hail.

Since my farm is heavily mortgaged, I badly need the money. My ife has been sick all summer. If you do not pay on my claim at once I hall sue the company".

The company still refused to pay, claiming that there was no hail.

irections: Using only the above data and the principles of logical easoning with which you are familiar, select one of the following onclusions.

NCLUSIONS:

The company should pay the insurance.
The company should not pay the insurance.
More information is required before we can decide whether or not the company should pay the insurance.

irections: Choose the reasons which support or explain the logic of your conclusion.

ASONS:

The word "hail" must be accurately defined.
The Granham Times reported snow and one can surely rely on anything that appears in print.
The farmer was a fool to think that just because he said it was hail, the company would pay the claim.
A rich insurance company should certainly pay a poor farmer so obviously in need as Mr. Metz was.
A changed definition changes the conclusion even though the argument from each definition may be logical.
The insurance company was merely trying to get out of paying the claim by stating that there was no hail.
Mr. Metz is assuming the conclusion when he says that the storm was hail.
Since Mr. Metz in his letter as quoted, claimed that the local teacher, the minister and several other prominent citizens could testify that the storm was hail, it probably was hail.
In a sound argument, one cannot use the conclusion one set out to prove as a reason to support the argument.
If there was a hailstorm then the company should pay the insurance.
Just as rain in winter usually turns to snow, a hailstorm in summer ends up with snow, then rain; so the storm was hail.
The conclusion is not necessarily true, but if we accept the premises on which an argument is based, to be logical we should accept the conclusion which logically follows.

oblem 10.

Jane summed up her report on "Democracy as a Way of Life" to the cial Studies class as follows:

"I have produced evidence to show that there are really only three nds of government in the world today: communist, fascist and democratic. have shown that conditions in Russia and Germany, with communist and scist governments respectively, do not satisfy the needs of the common ople. I also demonstrated that in all other fascist or communist govern- countries, the common people are not satisfied. Hence, democracy is e only form of government which satisfies the common people."

irections: Let us assume that Jane provided convincing facts to show hat conditions in all fascist and communist governed countries are un- atisfactory for the common people. Let us also assume that according to he definition she used, there are only three types of government in the orld today (communist, fascist, democratic). Using only these data and he principles of logical reasoning with which you are familiar, choose ne of the following conclusions.

NCLUSIONS.

Jane did not prove that democracy is the only form of government which satisfies the common people.

Jane proved that democracy is the only form of government which satis- fies the common people.

There is not sufficient information given to decide whether Jane did or did not prove that democracy is the only form of government which satisfies the common people.

ections: Choose the reasons which support or explain the logic of your conclusion.

SONS:

We must know how Jane defined "communist, fascist and democratic" before we can decide whether she proved her point.

The only thing Jane proved was that the saying "Fools rush in where angels fear to tread" applies to her.

Goebbels claims that the common people of Germany are satisfied, and we require more information to know whether his claims are correct.

Since we are told to assume that Jane proved that conditions in all fascist governed and communist governed countries are unsatisfactory for the common people, and since democratic government is the only one left, then democracy must be the only form of government which satisfies the common people.

In an indirect argument one must eliminate all the possibilities except one and then accept the remaining possibility, as it stands, as the conclusion.

Jane's arguments were silly because she wanted to please her teacher and she knew she must favour democracy as a way of life to do that.

The conclusion may or may not be true, but if we accept the premises on which it is based, to be logical we must accept the conclusion which logically follows.

changed definition leads to a changed conclusion even though the argu- ment from each is logical.

ane showed that fascist and communist governments are unsatisfactory to he common people, which left democracy as the only remaining kind of overnment, but she did not show that it satisfies the common people.

ane's father said just the night before Jane gave the report that the eople of Russia and Germany were on the verge of open revolt.

school in which the teacher keeps good discipline is better ordered han one where students run wild. Similarly, the good order kept by ascist or communist governments satisfies the common people better han a democratic government does.

ccording to the statements Jane made in the summary of her report there ight be many more than just three kinds of governments in the world today.

ANSWER SHEET

Name: ________________________ School: ________________________

Age: _____ (yrs.) _____ (months) Sex: ________ City or town: ____________________

Grade: __________ Province: ____________________

Number of years you have been attending High School (including this year): ________

Geometry teacher: ____________________

Number of courses in Demonstrative Geometry you have taken (including courses you are taking this year): ________

Your standing in class in geometry course you are now taking (Check):

_____ highest 1/3 _____ middle 1/3 _____ lowest 1/3

	Conclusion			REASONS																	
	A	B	C	1	2	3	4	5	6	7	8	9	10	11	12						
Sample Problem	X					X				X											
Problem 1.																					
Problem 2.																					
Problem 3.																					
Problem 4.																					
Problem 5.																					
Problem 6.																					
Problem 7.																					
Problem 8.																					
Problem 9.																					
Problem 10.																					